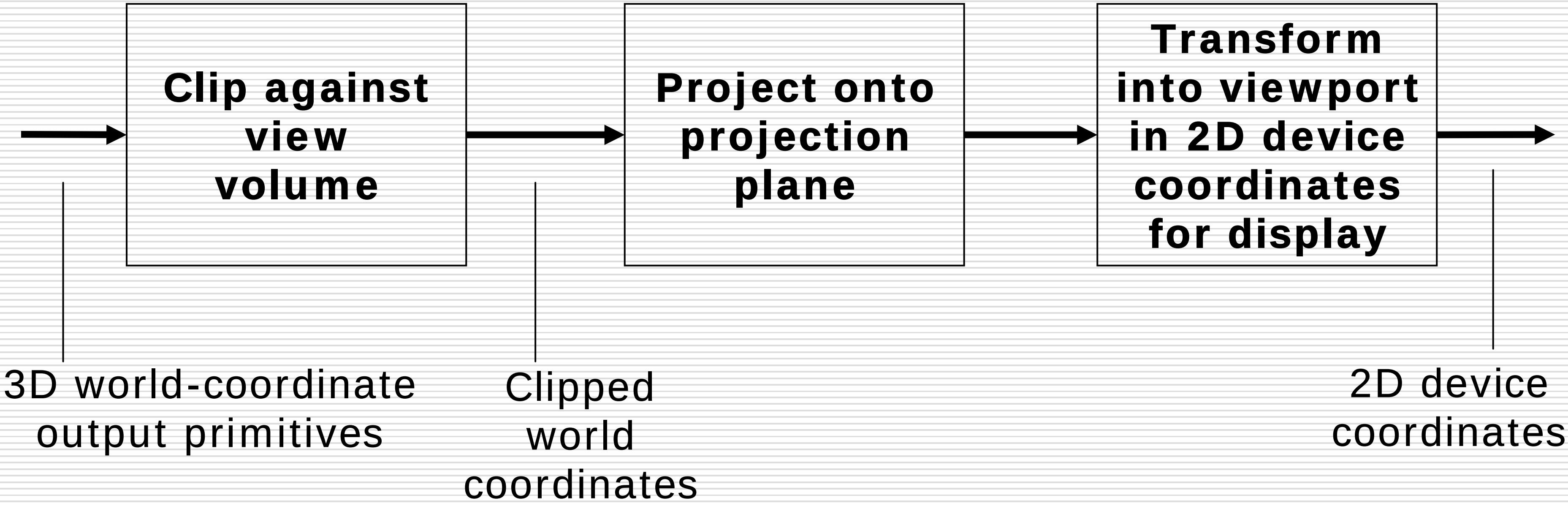
Computer Graphics

Jeng-Sheng Yeh 葉正聖
Ming Chuan University
(modified from Bing-Yu Chen’s slides)

Viewing in 3D

- 3D Viewing Process
- Specification of an Arbitrary 3D View
- Orthographic Parallel Projection
- Perspective Projection
- 3D Clipping for Canonical View Volume

3D Viewing Process



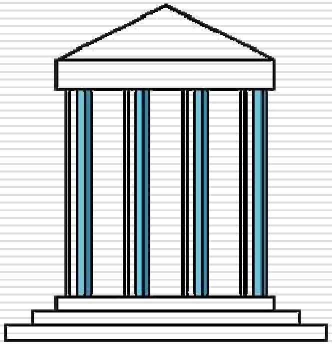
Classical Viewing

- Viewing requires three basic elements
  - One or more objects
  - A viewer with a projection surface
  - Projectors that go from the object(s) to the projection surface
- Classical views are based on the relationship among these elements
  - The viewer picks up the object and orients it how she would like to see it
- Each object is assumed to constructed from flat *principal faces*
  - Buildings, polyhedra, manufactured objects

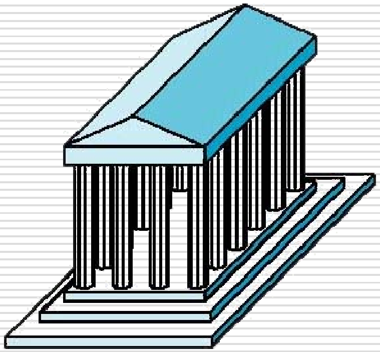
Planar Geometric Projections

- Standard projections project onto a plane
- Projectors are lines that either
  - converge at a center of projection are parallel
- Such projections preserve lines
  - but not necessarily angles
- Nonplanar projections are needed for applications such as map construction

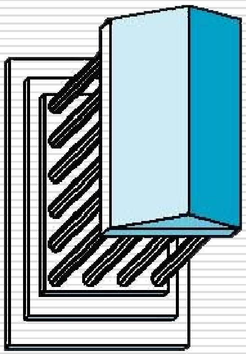
Classical Projections



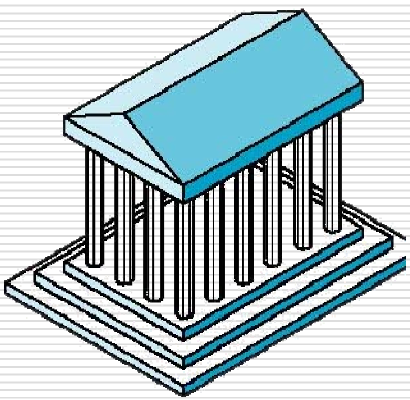
Front elevation



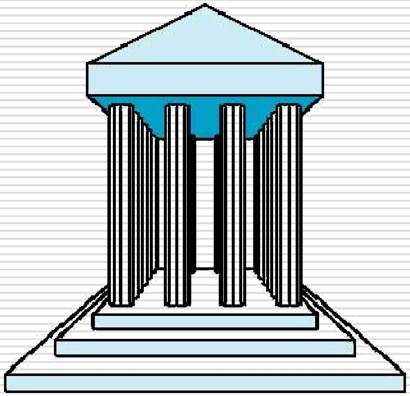
Elevation oblique



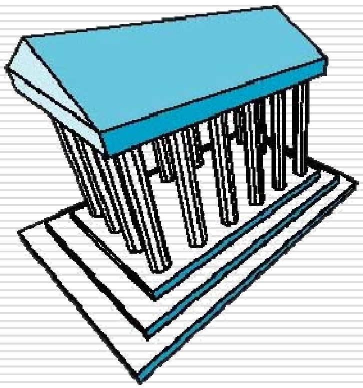
Plan oblique



Isometric



One-point perspective

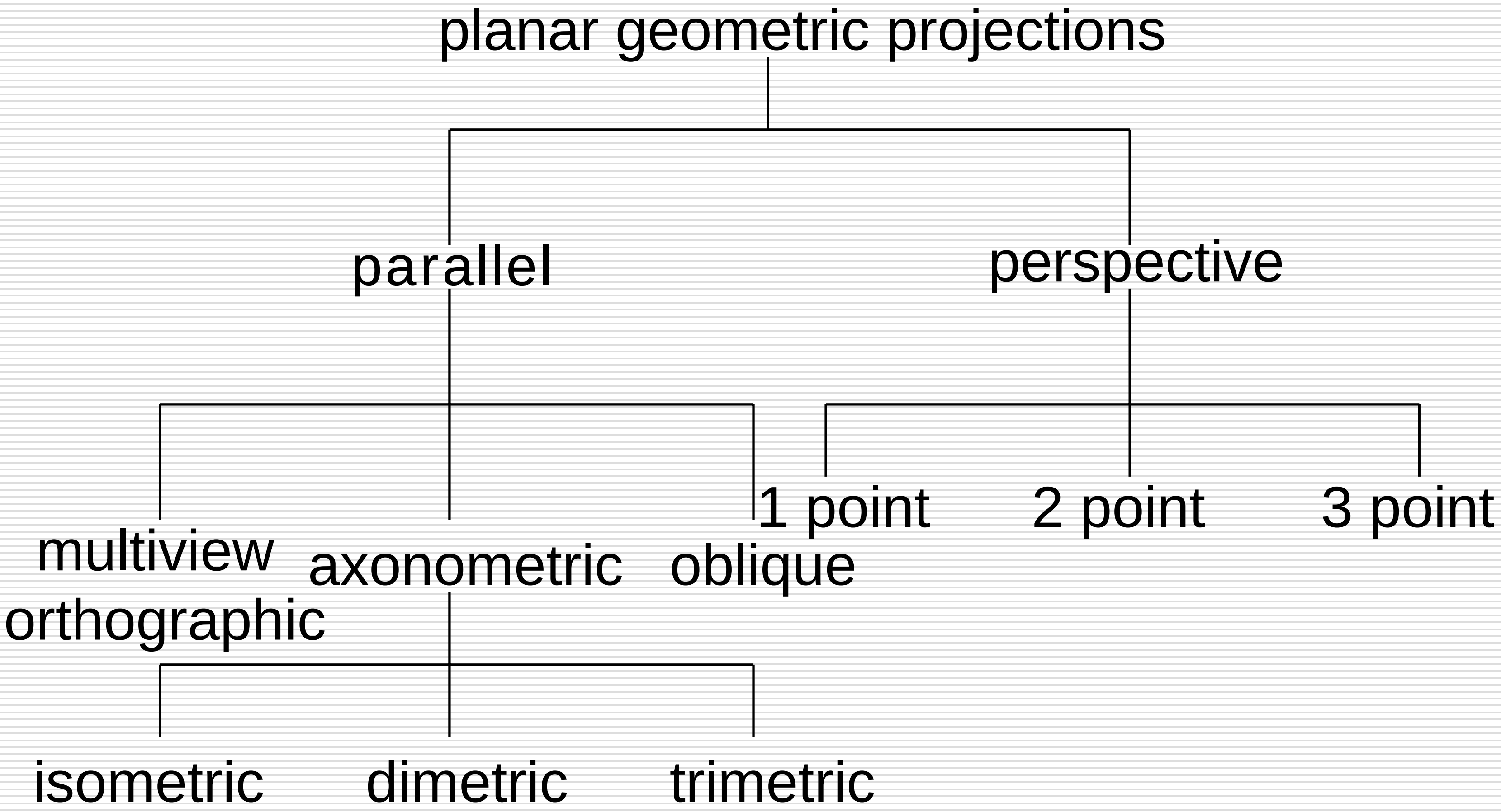


Three-point perspective

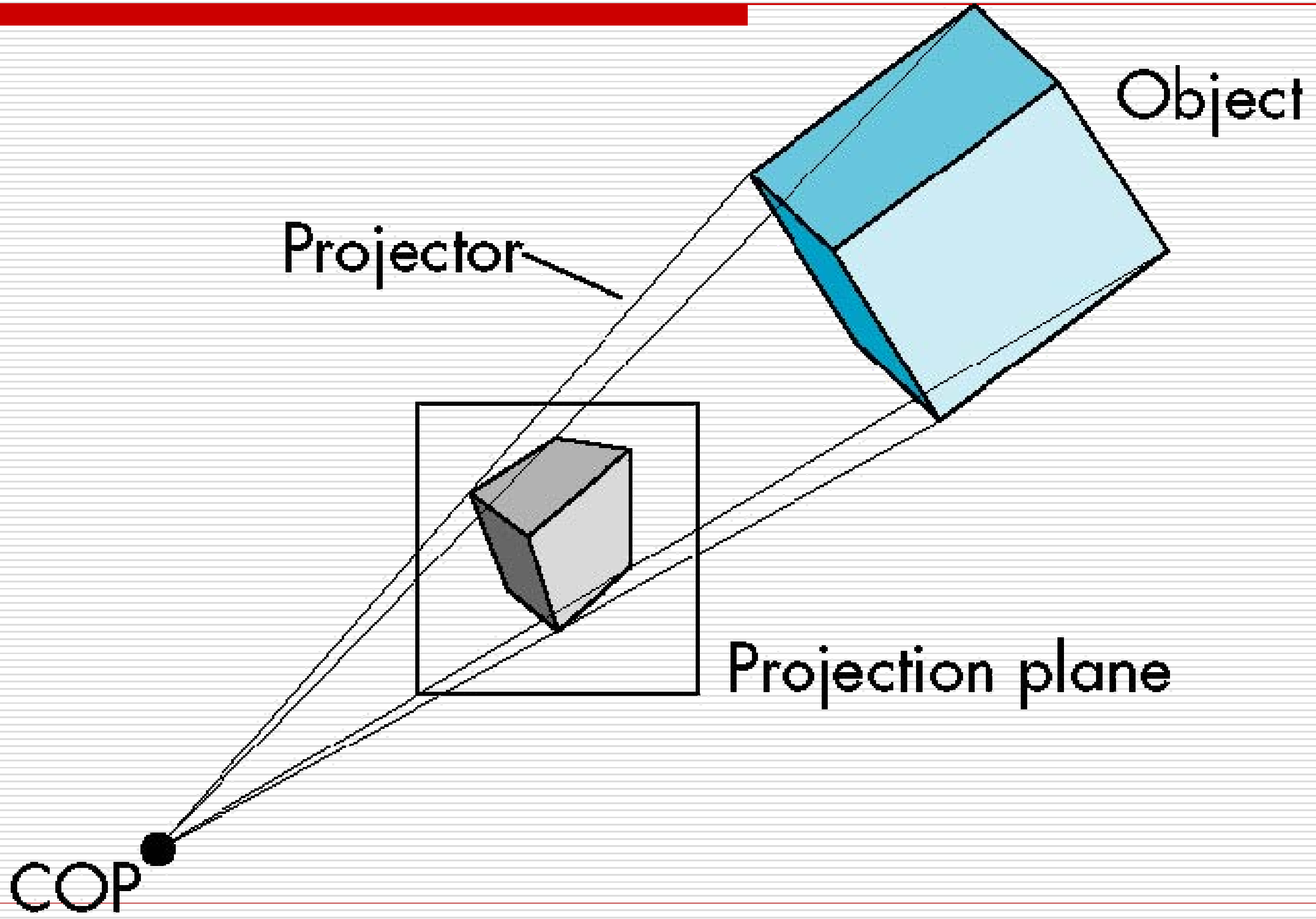
Perspective vs. Parallel

- Computer graphics treats all projections the same and implements them with a single pipeline
- Classical viewing developed different techniques for drawing each type of projection
- Fundamental distinction is between parallel and perspective viewing even though mathematically parallel viewing is the limit of perspective viewing

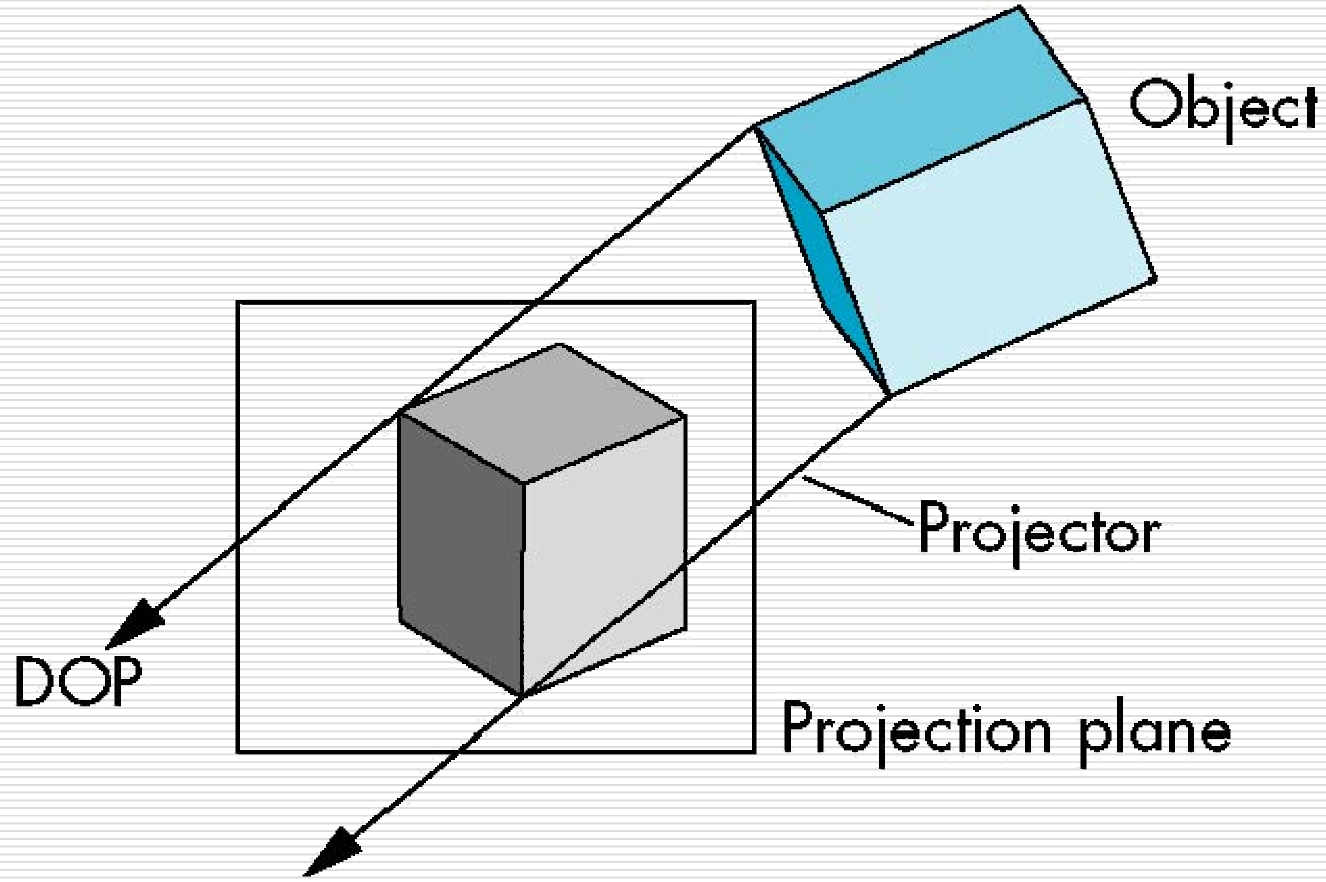
Taxonomy of Planar Geometric Projections



Perspective Projection

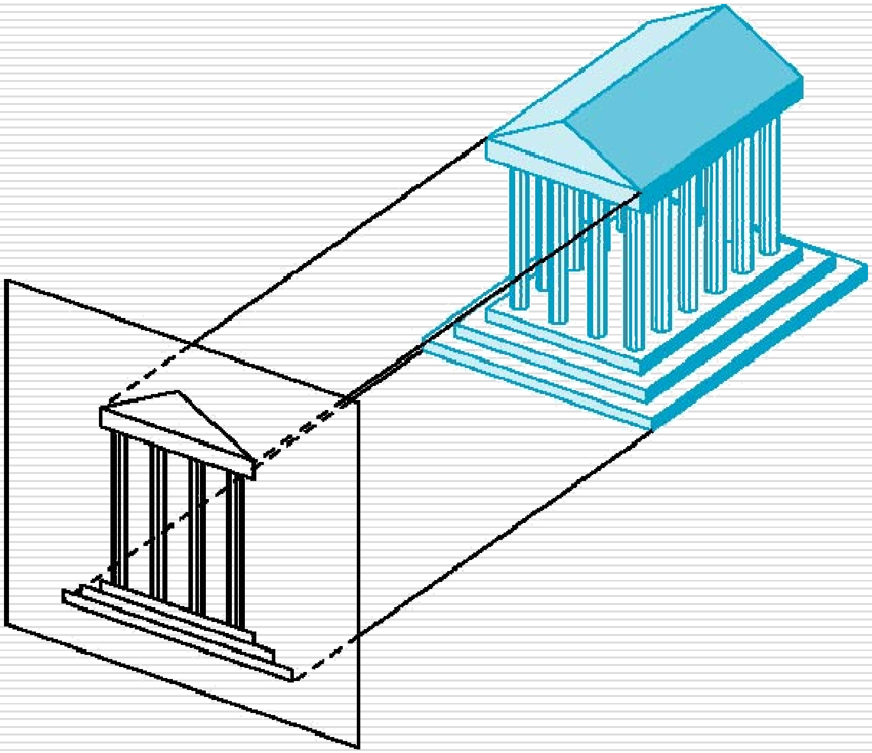


Parallel Projection



Orthographic Projection

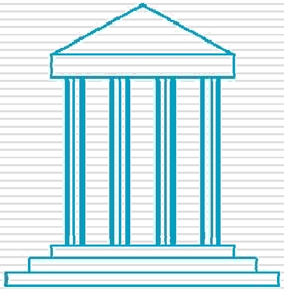
Projectors are orthogonal to projection surface



Multiview Orthographic Projection

- Projection plane parallel to principal face
- Usually form front, top, side views

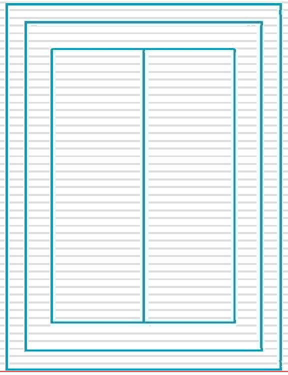
isometric (not multiview orthographic view)

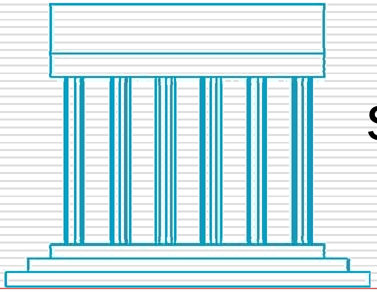


front

in CAD and architecture, we often display three multiviews plus isometric

top





side

Advantages and Disadvantages

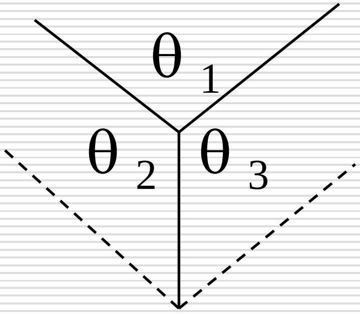
- Preserves both distances and angles
  - Shapes preserved
  - Can be used for measurements
    - Building plans
    - Manuals
- Cannot see what object really looks like because many surfaces hidden from view
  - Often we add the isometric

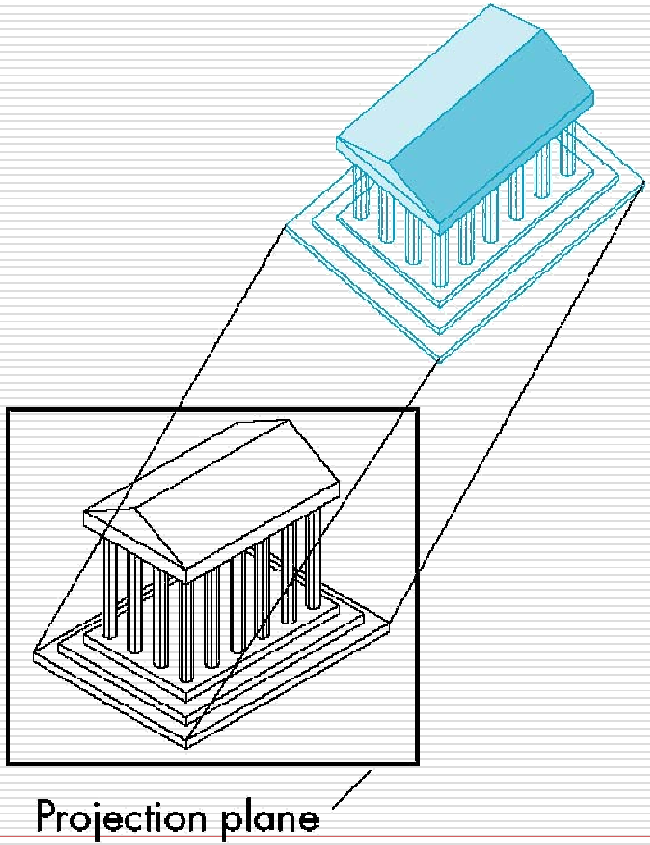
Axonometric Projections

Allow projection plane to move relative to object

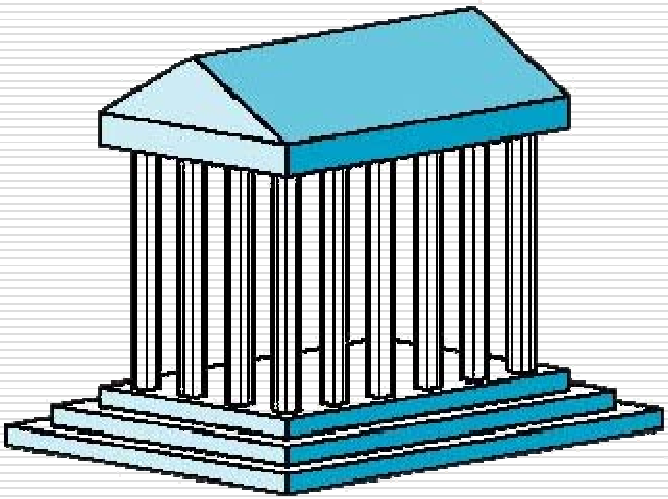
classify by how many angles of a corner of a projected cube are the same

none: trimetric
two: dimetric
three: isometric

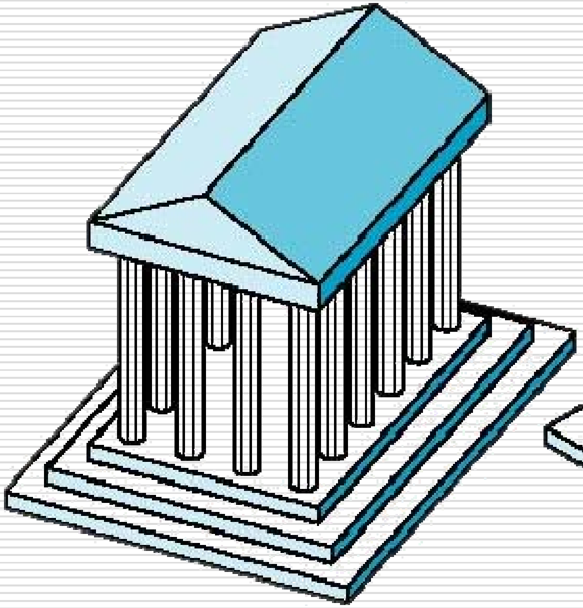




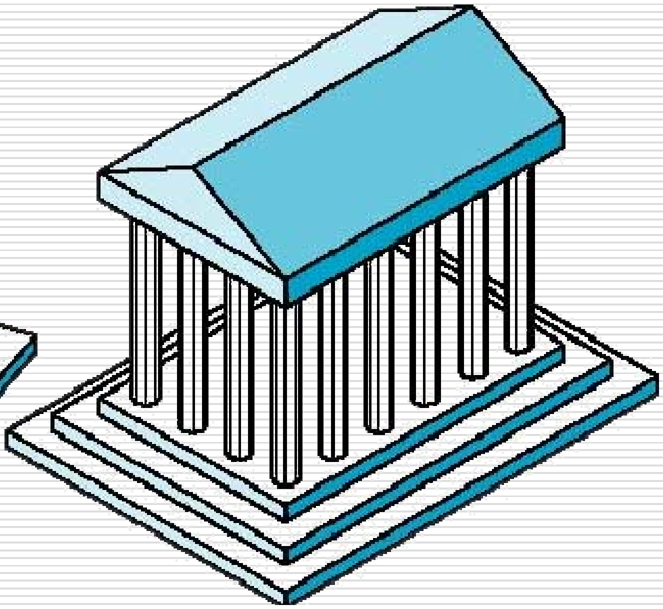
Types of Axonometric Projections



Dimetric



Trimetric



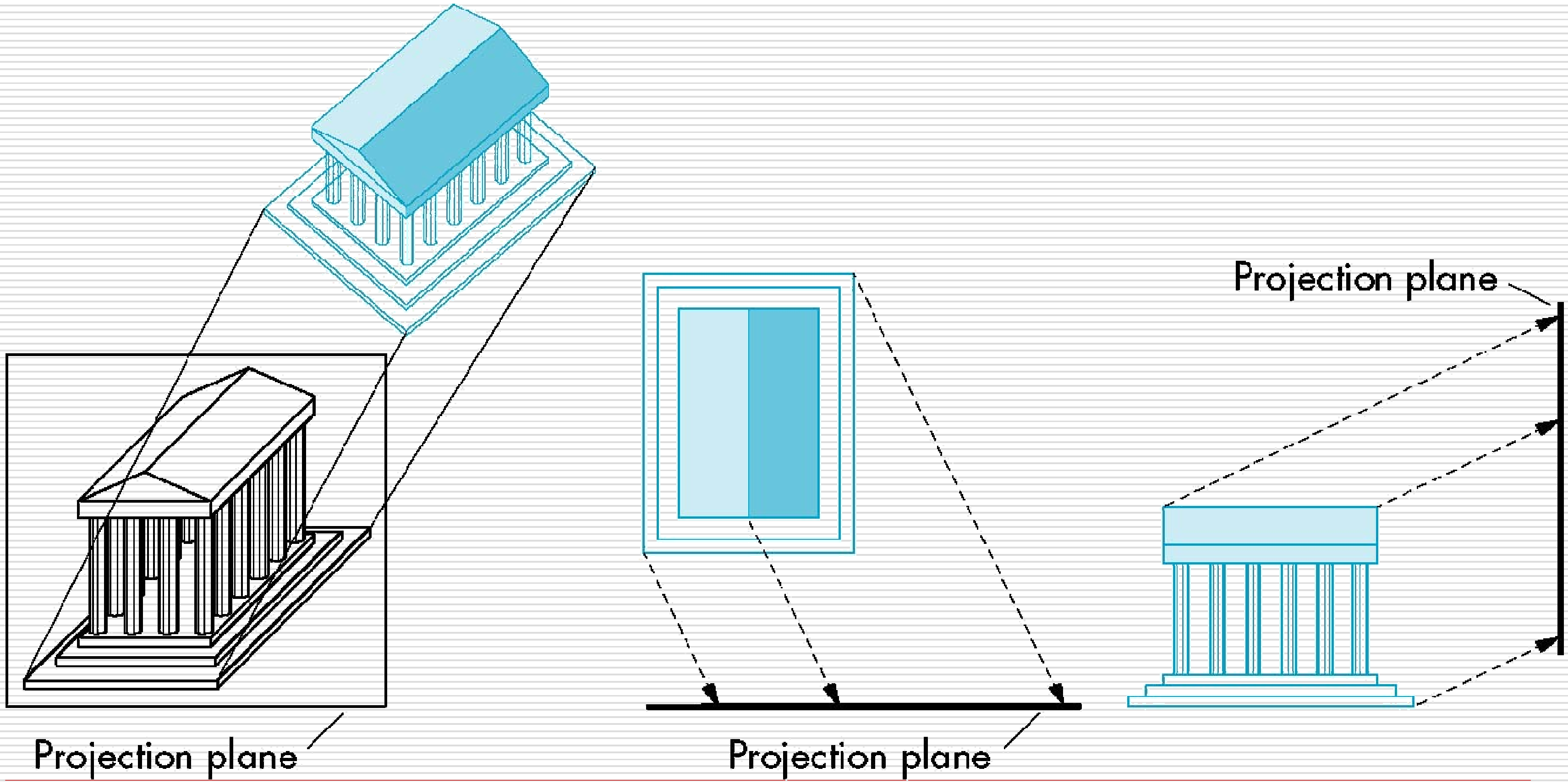
Isometric

Advantages and Disadvantages

- Lines are scaled (*foreshortened*) but can find scaling factors
- Lines preserved but angles are not
  - Projection of a circle in a plane not parallel to the projection plane is an ellipse
- Can see three principal faces of a box-like object
- Some optical illusions possible
  - Parallel lines appear to diverge
- Does not look real because far objects are scaled the same as near objects
- Used in CAD applications

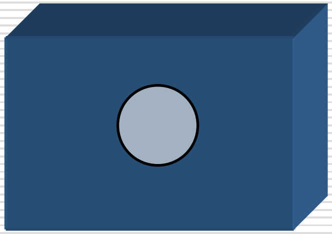
Oblique Projection

Arbitrary relationship between projectors and projection plane



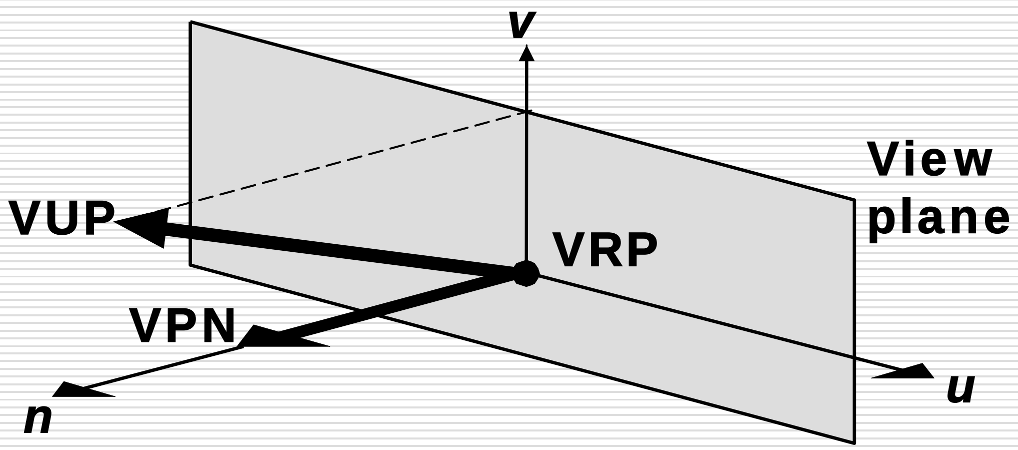
Advantages and Disadvantages

- Can pick the angles to emphasize a particular face
  - Architecture: plan oblique, elevation oblique
- Angles in faces parallel to projection plane are preserved while we can still see “around” side



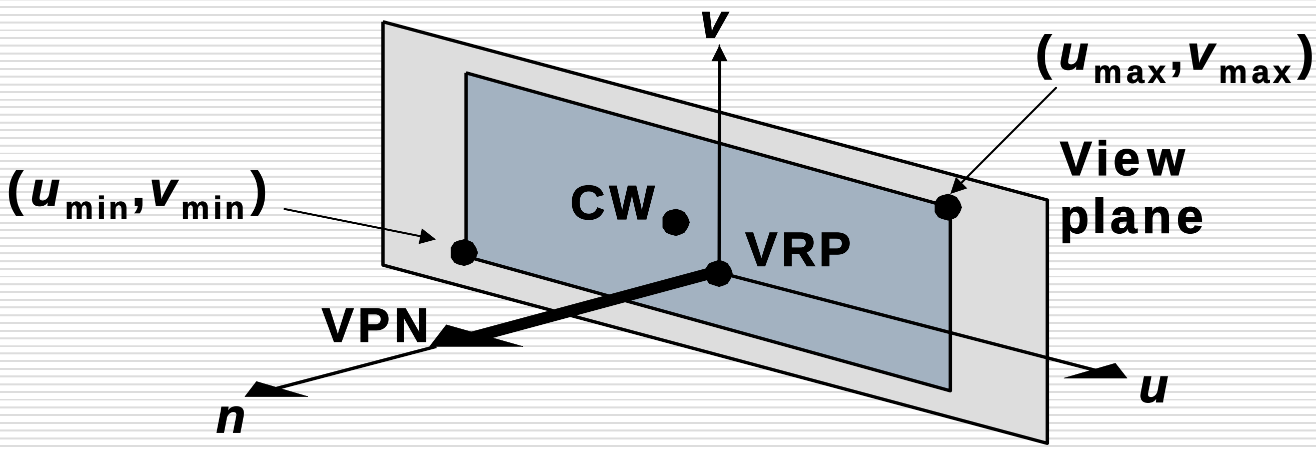
- In physical world, cannot create with simple camera; possible with bellows camera or special lens (architectural)

Specification of an Arbitrary 3D View



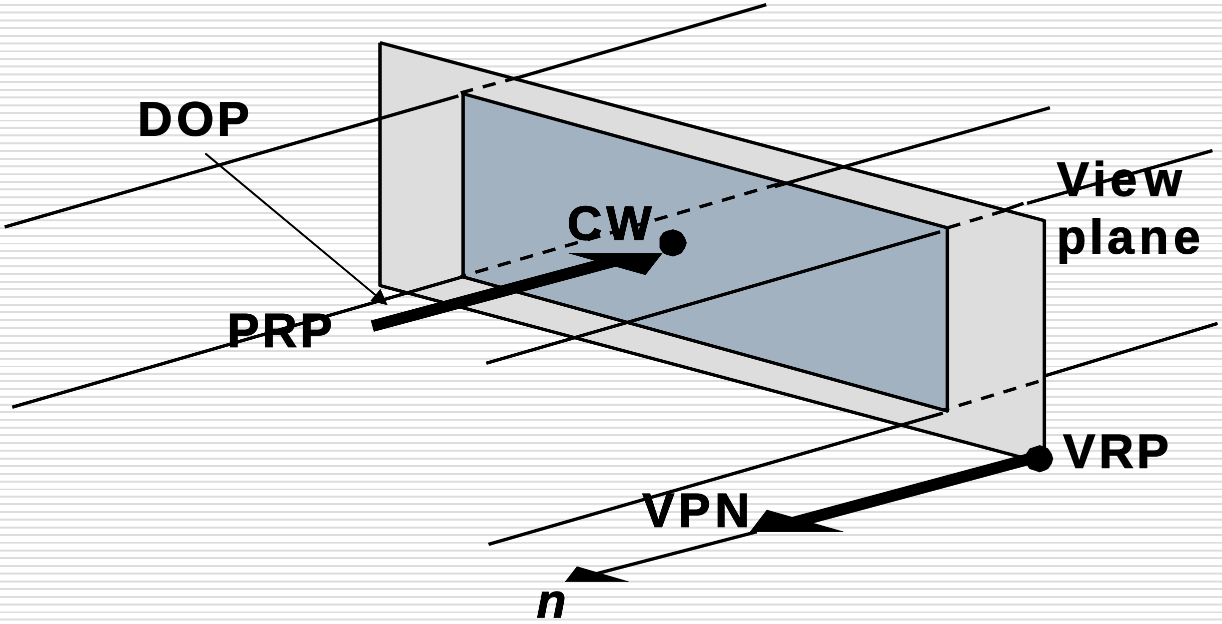
- VRP: view reference point
- VPN: view-plane normal
- VUP: view-up vector

VRC:
the viewing-reference coordinate system



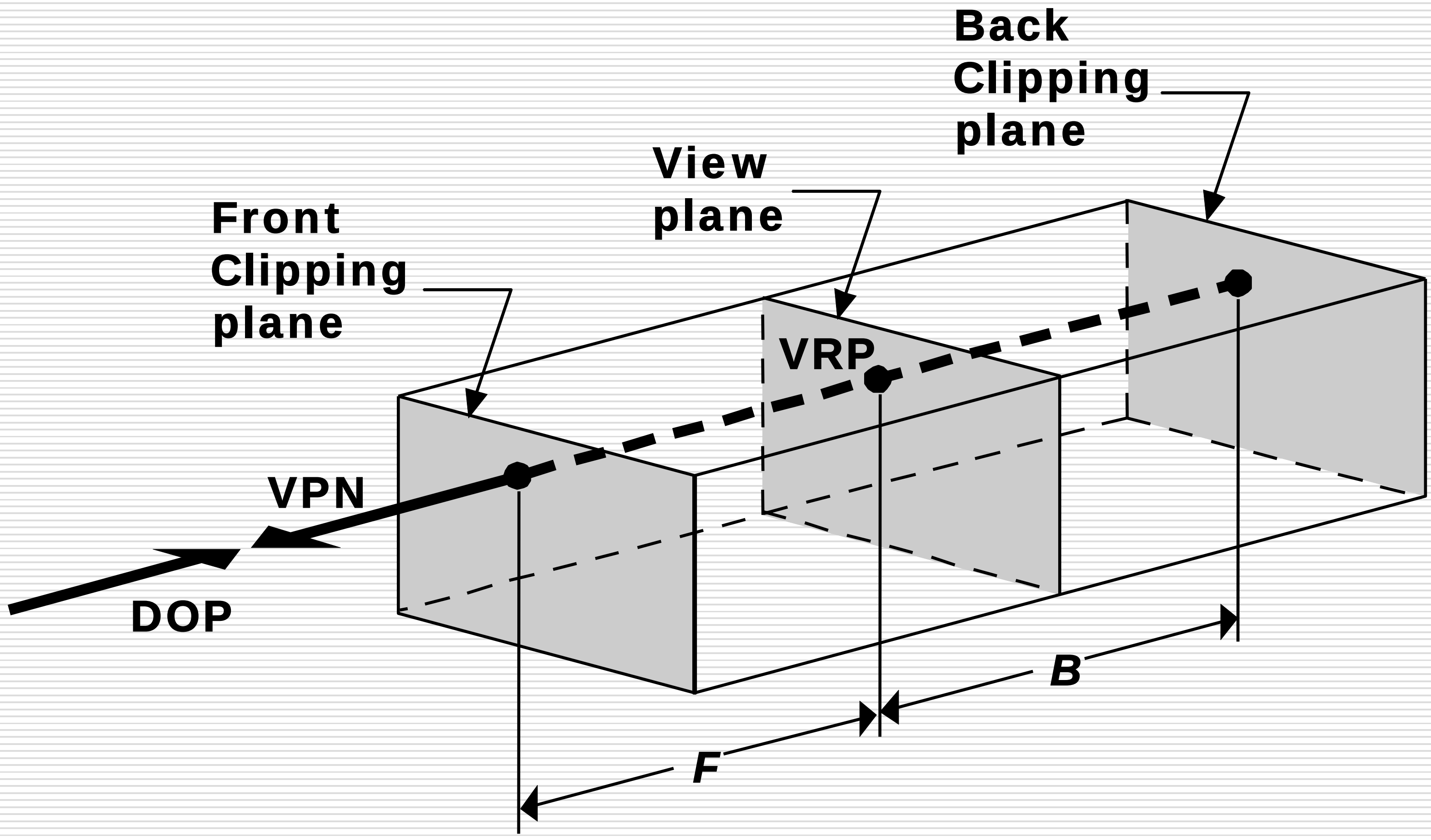
- CW: center of the window

Infinite Parallelepiped View Volume

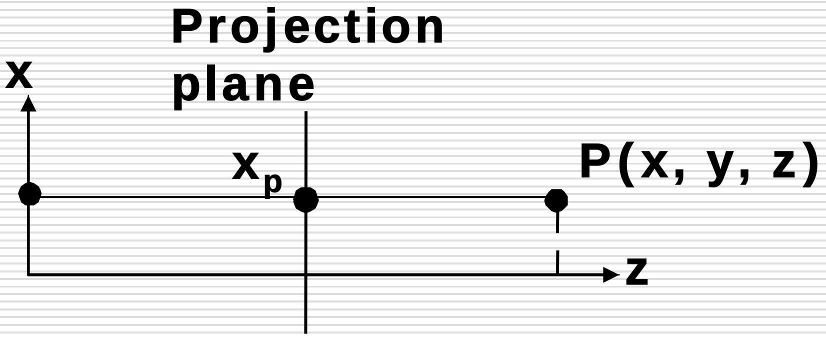


- DOP: direction of projection
- PRP: projection reference point

Truncated View Volume for an Orthographic Parallel Projection

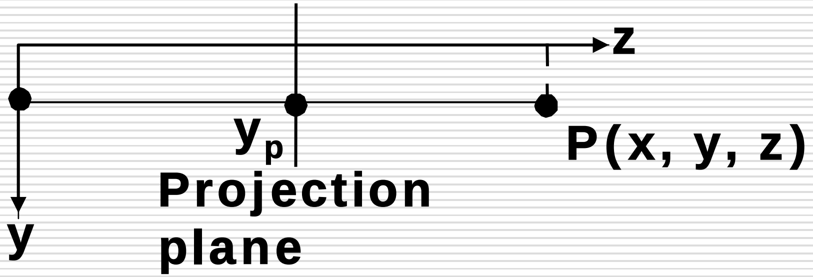


The Mathematics of Orthographic Parallel Projection



View along y axis

View along x axis



$$x_p = x; y_p = y; z_p = 0$$

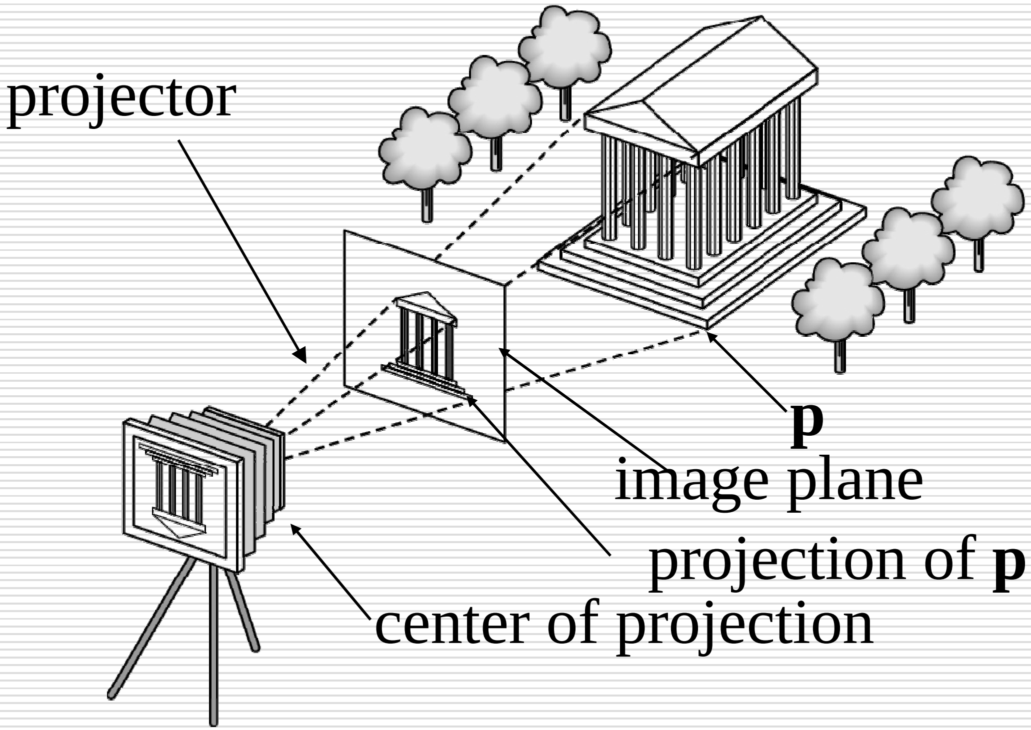
$$M_{ort} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The Steps of Implementation of Orthographic Parallel Projection

- Translate the VRP to the origin
- Rotate VRC such that the VPN becomes the z axis
- Shear such that the DOP becomes parallel to the z axis
- Translate and scale into the parallel-projection canonical view volume

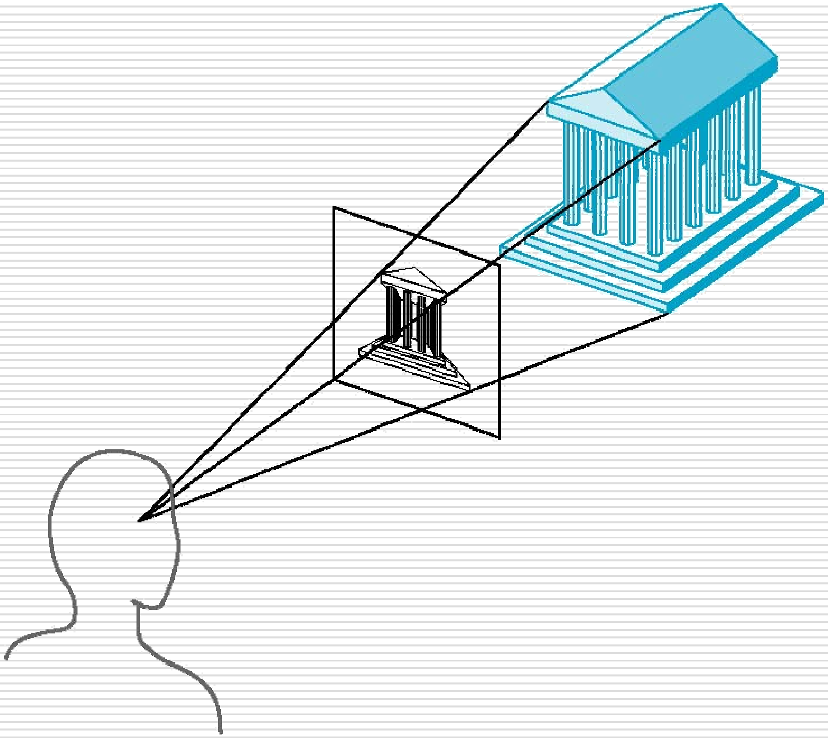
$$N_{par} = S_{par} \bullet T_{par} \bullet SH_{par} \bullet R \bullet T(-VRP)$$

Synthetic Camera Model

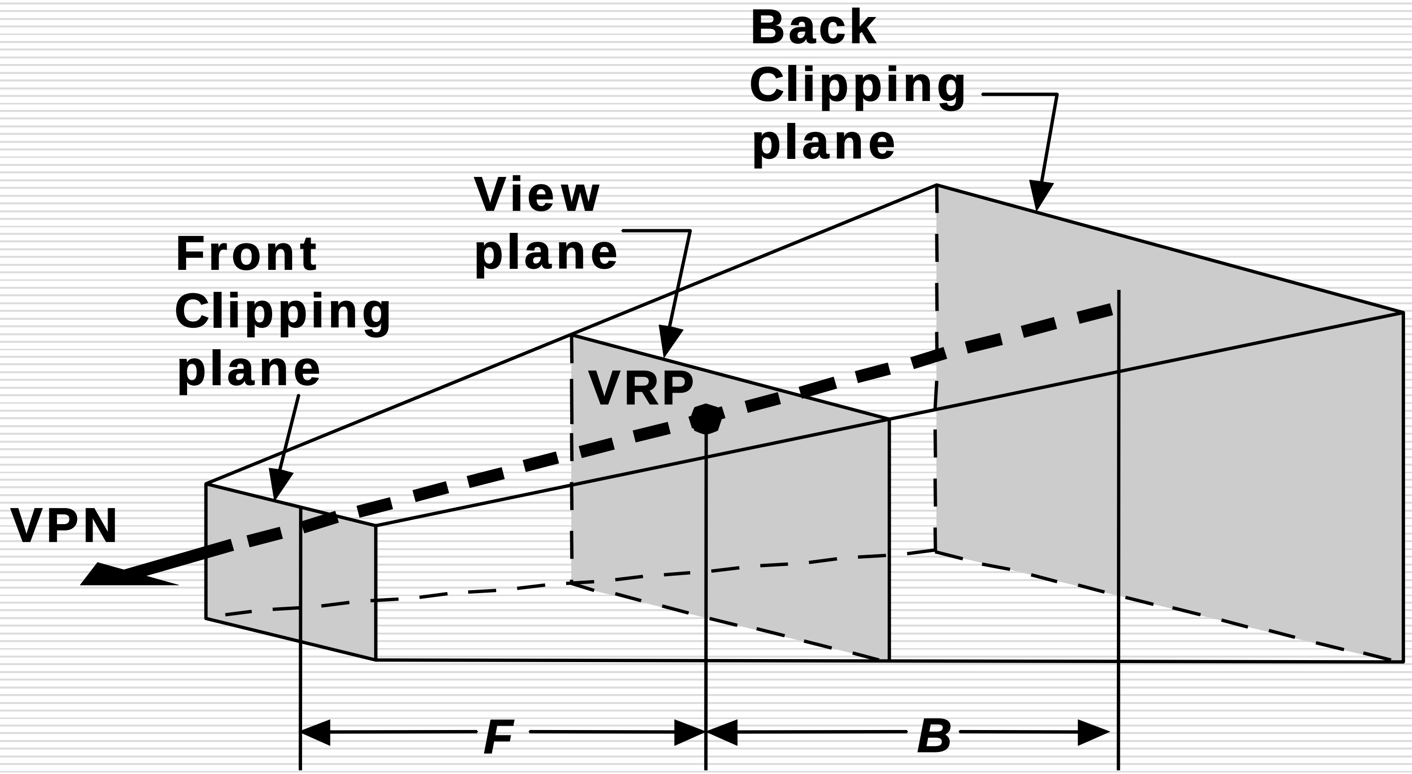


Perspective Projection

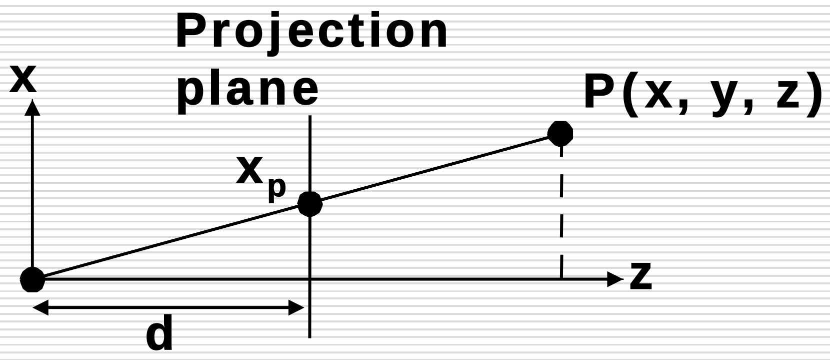
Projectors coverge at center of projection



Truncated View Volume for an Perspective Projection

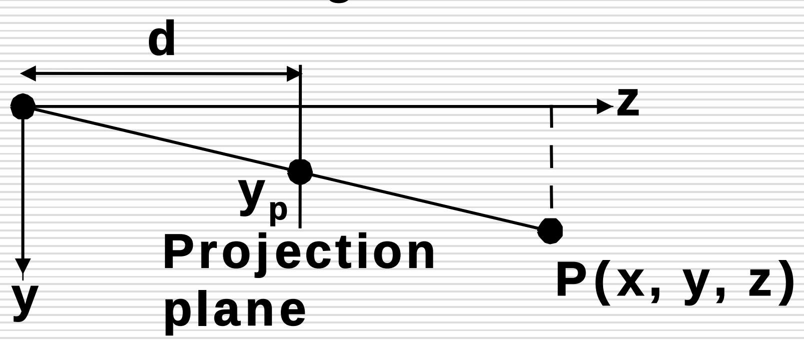


Perspective Projection (Pinhole Camera)



View along y axis

View along x axis



$$\frac{x_p}{d} = \frac{x}{z}; \frac{y_p}{d} = \frac{y}{z}$$

$$x_p = \frac{x}{z/d}; y_p = \frac{y}{z/d}$$

$$M_{per} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/d & 0 \end{bmatrix}$$

Perspective Division

$$\begin{bmatrix} x_p \\ y_p \\ z_p \\ 1 \end{bmatrix} = \begin{bmatrix} X \\ Y \\ Z \\ W \end{bmatrix} = M_{per} \bullet P = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/d & 0 \end{bmatrix} \bullet \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \\ \frac{z}{d} \end{bmatrix}$$

However $W \neq 1$, so we must divide by W to return from homogeneous coordinates

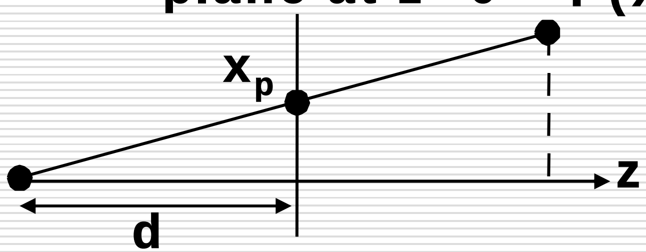
$$\left(x_p, y_p, z_p\right) = \left(\frac{X}{W}, \frac{Y}{W}, \frac{Z}{W}\right) = \left(\frac{x}{z/d}, \frac{y}{z/d}, d\right)$$

The Steps of Implementation of Perspective Projection

- Translate the VRP to the origin
- Rotate VRC such that the VPN becomes the z axis
- Translate such that the PRP is at the origin
- Shear such that the DOP becomes parallel to the z axis
- Scale such that the view volume becomes the canonical perspective view volume

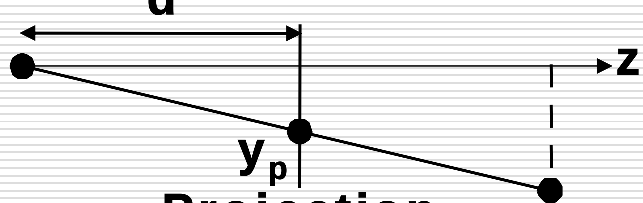
$$N_{per} = S_{per} \bullet SH_{per} \bullet T(-PRP) \bullet R \bullet T(-VRP)$$

Alternative Perspective Projection



View along y axis

View along x axis



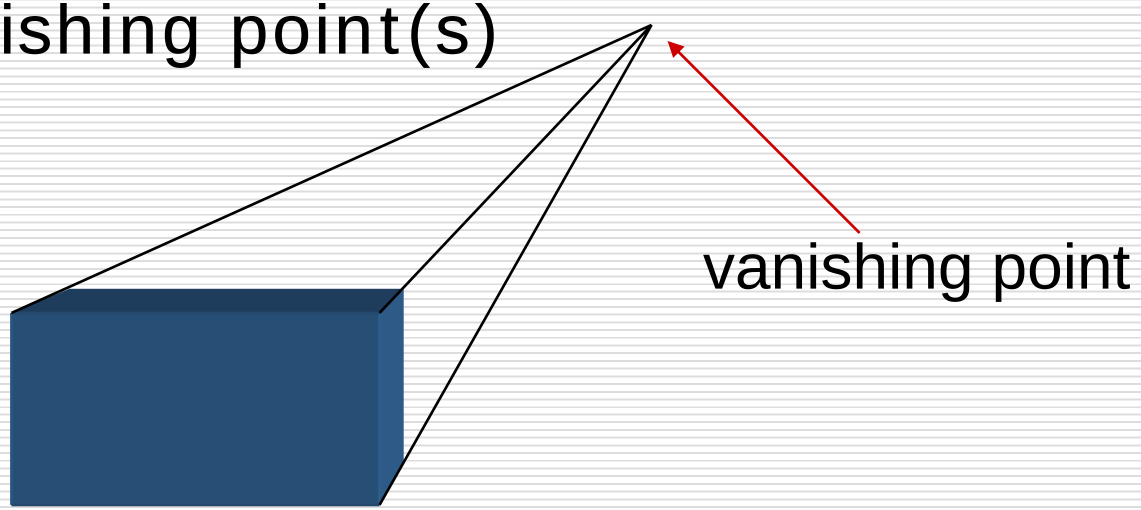
$$\frac{x_p}{d} = \frac{x}{z+d}; \frac{y_p}{d} = \frac{y}{z+d}$$

$$x_p = \frac{x}{(z/d)+1}; y_p = \frac{y}{(z/d)+1}$$

$$M'_{per} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1/d & 1 \end{bmatrix}$$

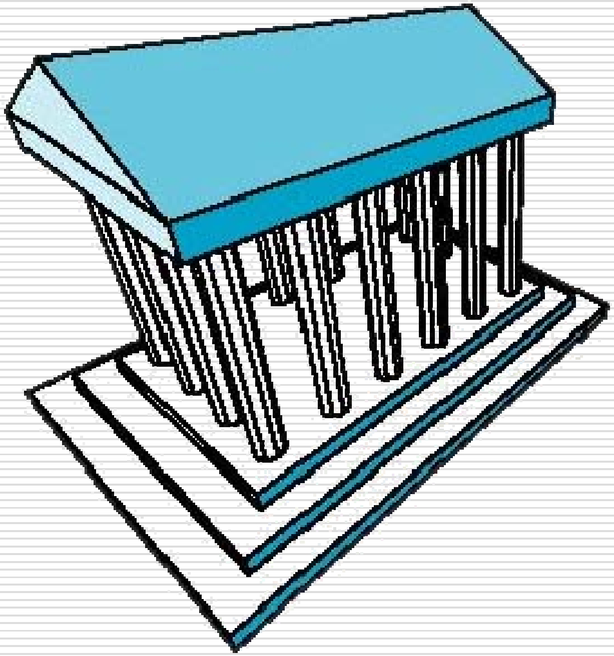
Vanishing Points

- Parallel lines (not parallel to the projection plan) on the object converge at a single point in the projection (the *vanishing point*)
- Drawing simple perspectives by hand uses these vanishing point(s)



Three-Point Perspective

- No principal face parallel to projection plane
- Three vanishing points for cube

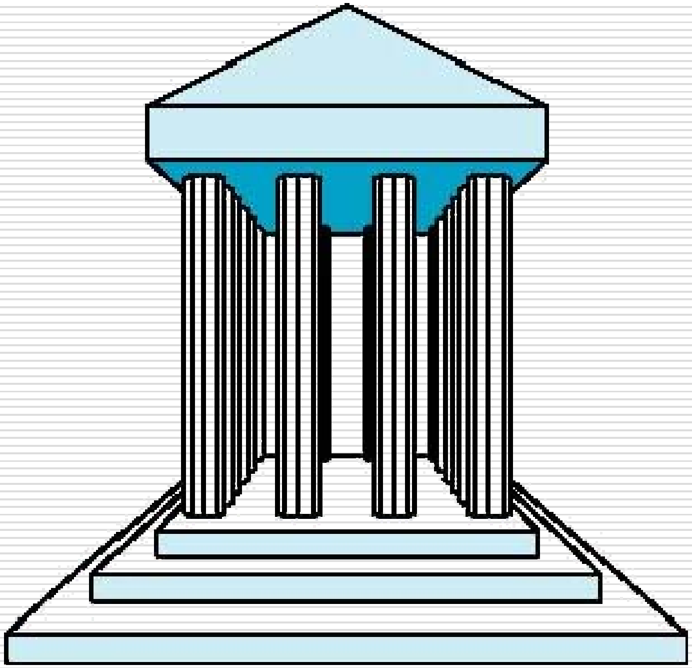


Two-Point Perspective

- On principal direction parallel to projection plane
- Two vanishing points for cube

One-Point Perspective

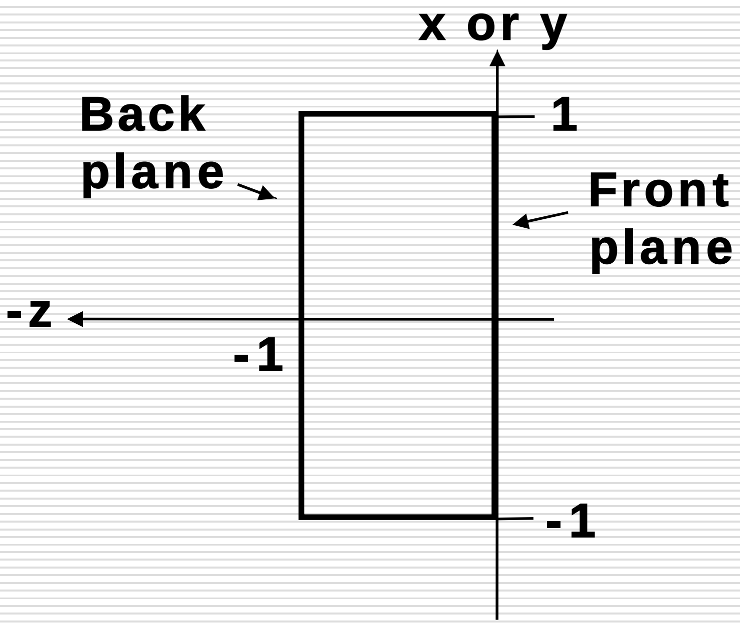
- One principal face parallel to projection plane
- One vanishing point for cube



Advantages and Disadvantages

- Objects further from viewer are projected smaller than the same sized objects closer to the viewer (*diminuition*)
  - Looks realistic
- Equal distances along a line are not projected into equal distances (*nonuniform foreshortening*)
- Angles preserved only in planes parallel to the projection plane
- More difficult to construct by hand than parallel projections (but not more difficult by computer)

Canonical View Volume for Orthographic Parallel Projection



- x = -1, y = -1, z = 0
- x = 1, y = 1, z = -1

The Extension of the Cohen-Sutherland Algorithm

bit 1 – point is above view volume	$y > 1$
bit 2 – point is below view volume	$y < -1$
bit 3 – point is right of view volume	$x > 1$
bit 4 – point is left of view volume	$x < -1$
bit 5 – point is behind view volume	$z < -1$
bit 6 – point is in front of view volume	$z > 0$

Intersection of a 3D Line

- a line from $P_0(x_0, y_0, z_0)$ to $P_1(x_1, y_1, z_1)$ can be represented as $x = x_0 + t(x_1 - x_0)$

$$y = y_0 + t(y_1 - y_0)$$

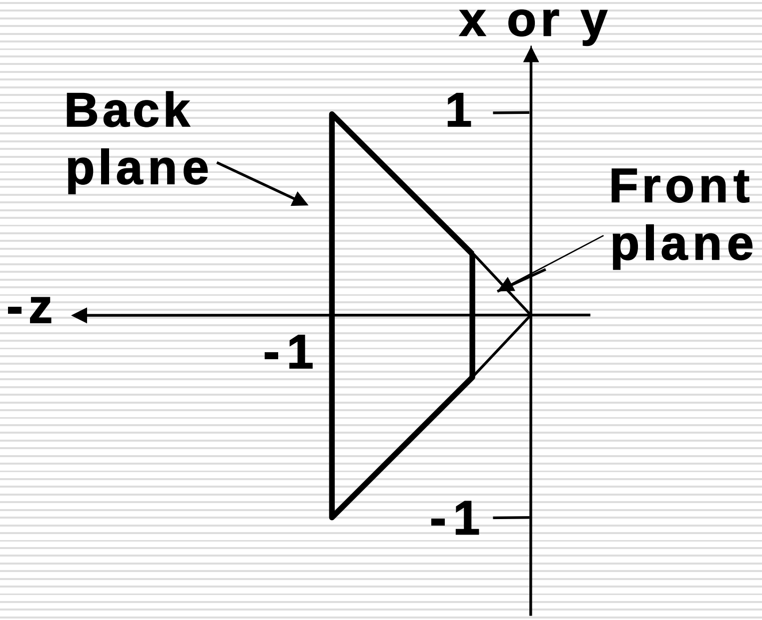
$$z = z_0 + t(z_1 - z_0) \quad 0 \le t \le 1$$

- so when y = 1

$$x = x_0 + \frac{(1 - y_0)(x_1 - x_0)}{y_1 - y_0}$$

$$z = z_0 + \frac{(1 - y_0)(z_1 - z_0)}{y_1 - y_0}$$

Canonical View Volume for Perspective Projection



- $x = z$, $y = z$, $z = -z_{min}$
- $x = -z$, $y = -z$, $z = -1$

The Extension of the Cohen-Sutherland Algorithm

- bit 1 – point is above view volume $y > -z$
- bit 2 – point is below view volume $y < z$
- bit 3 – point is right of view volume $x > -z$
- bit 4 – point is left of view volume $x < z$
- bit 5 – point is behind view volume $z < -1$
- bit 6 – point is in front of view volume $z > z_{min}$

Intersection of a 3D Line

- so when y = z

$$x = x_0 + \frac{(x_1 - x_0)(z_0 - y_0)}{(y_1 - y_0) - (z_1 - z_0)}$$

$$y = y_0 + \frac{(y_1 - y_0)(z_0 - y_0)}{(y_1 - y_0) - (z_1 - z_0)}$$

$$z = y$$

Clipping in Homogeneous Coordinates

- Why clip in **homogeneous coordinates** ?
  - it is possible to transform the *perspective-projection canonical view volume* into the *parallel-projection canonical view volume*

$$M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \dfrac{1}{1+z_{\min}} & \dfrac{-z_{\min}}{1+z_{\min}} \\ 0 & 0 & -1 & 0 \end{bmatrix}, z_{\min} \neq -1$$

Clipping in Homogeneous Coordinates

- The corresponding plane equations are
  - X = -W
  - X = W
  - Y = -W
  - Y = W
  - Z = -W
  - Z = 0