Computer Graphics

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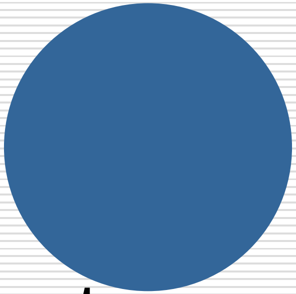
(modified from Bing-Yu Chen's slides)

Illumination and Shading

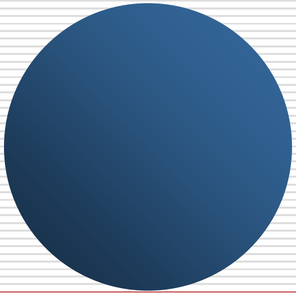
- Illumination Models
- Shading Models for Polygons
- Surface Detail
- Shadows
- Transparency
- Global Illumination
- Recursive Ray Tracing
- Radiosity
- The Rendering Pipeline

Why We Need Shading ?

- Suppose we build a model of a sphere using many polygons and color it with only one color. We get something like

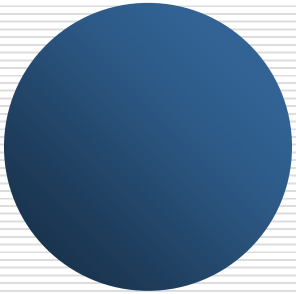


- But we want



Shading

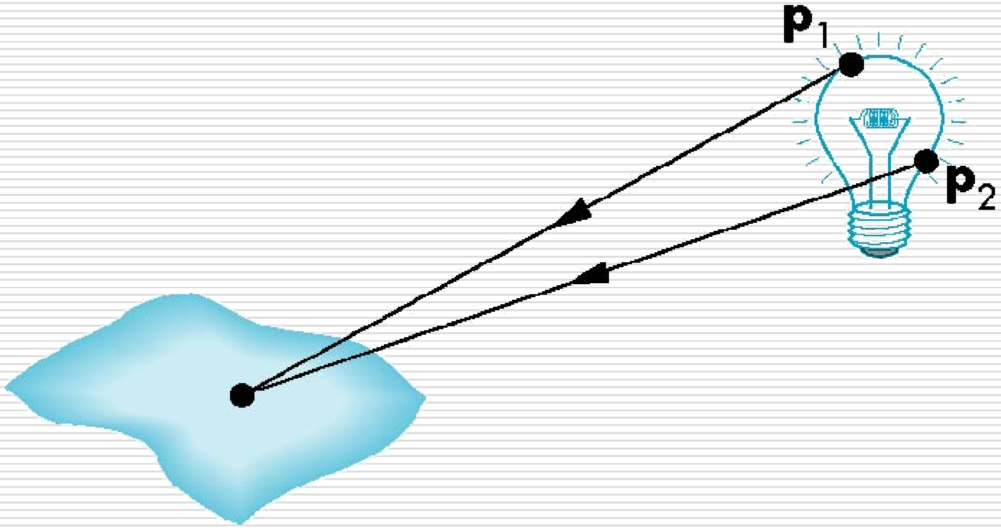
- Why does the image of a real sphere look like



- Light-material interactions cause each point to have a different color or shade
- Need to consider
  - Light sources
  - Material properties
  - Location of viewer
  - Surface orientation

Light Sources

- General light sources are difficult to work with because we must integrate light coming from all points on the source

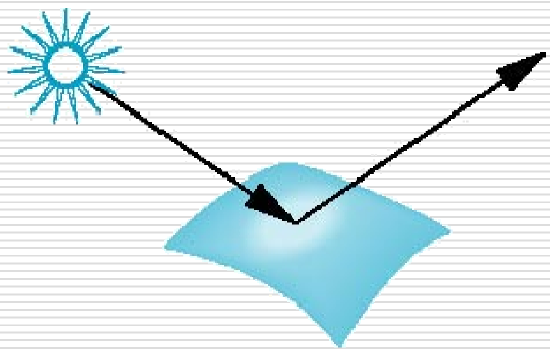


Simple Light Sources

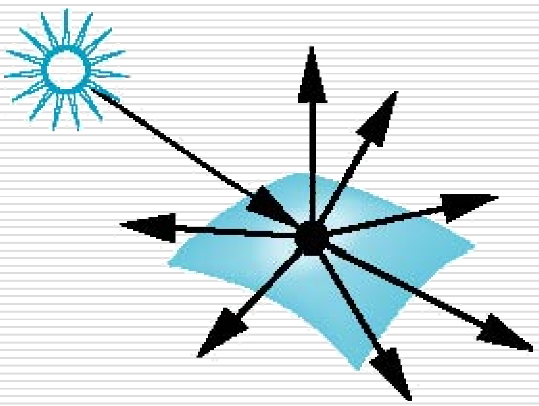
- Point source
  - Model with position and color
  - Distant source = infinite distance away (parallel)
- Spotlight
  - Restrict light from ideal point source
- Ambient light
  - Same amount of light everywhere in scene
  - Can model contribution of many sources and reflecting surfaces

Surface Types

- The smoother a surface, the more reflected light is concentrated in the direction a perfect mirror would reflected the light
- A very rough surface scatters light in all directions



smooth surface



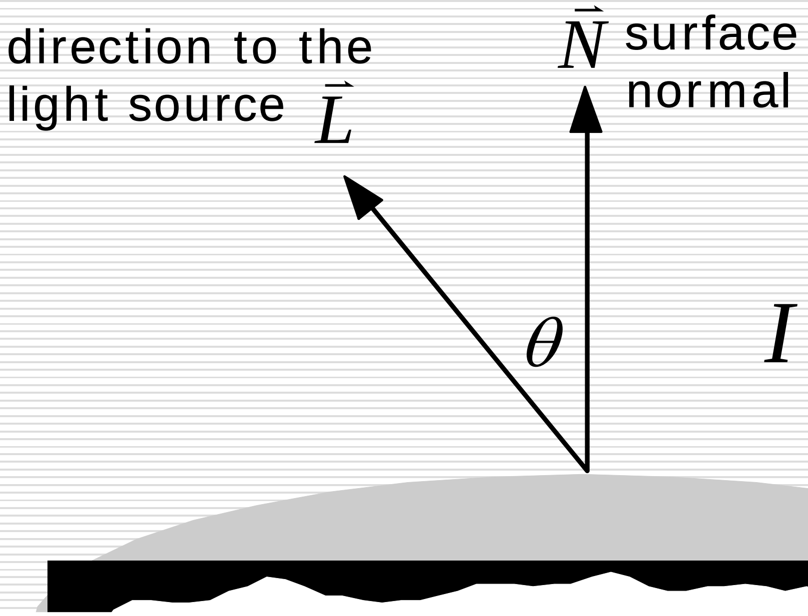
rough surface

Illumination Models

- Ambient Light: $I = I_a k_a$
  - I_a : intensity of the ambient light
  - k_a : ambient-reflection coefficient: 0 ~ 1

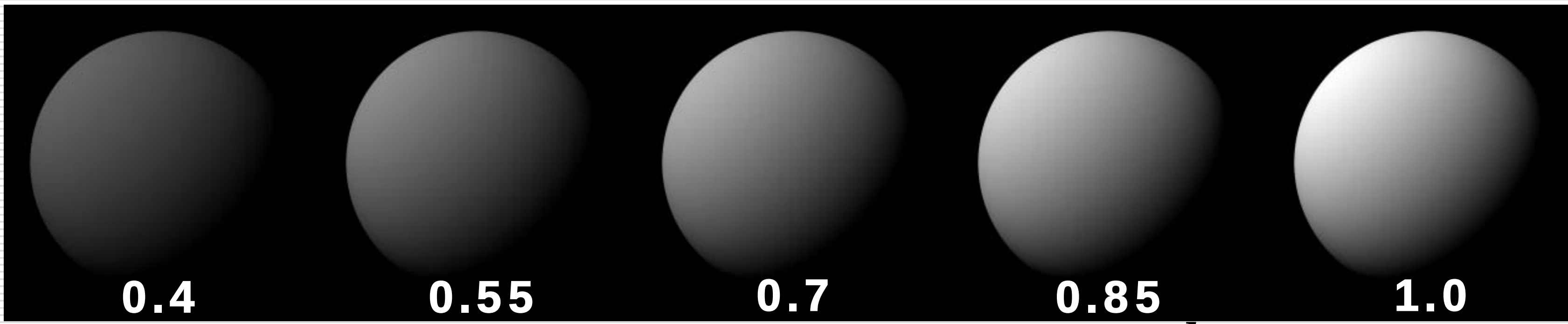
- Diffuse Reflection: $I = I_p k_d \cos\theta$
  - I_p : point light source's intensity
  - k_d : diffuse-reflection coefficient: 0 ~ 1
  - θ : angle: 0° ~ 90°

Diffuse Reflection

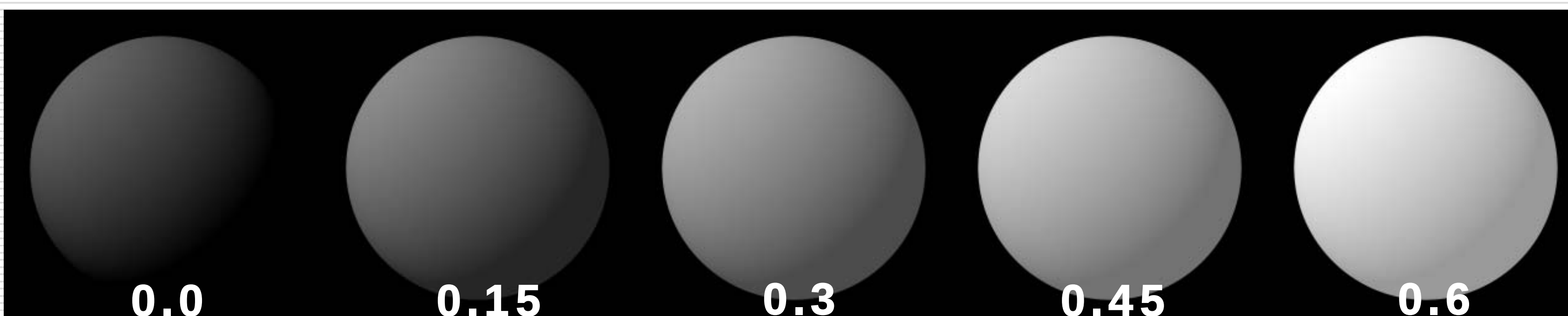


$$I = I_p k_d (\vec{N} \bullet \vec{L})$$

Examples



diffuse-reflection model with different k_d



ambient and diffuse-reflection model with different k_a
and $I_a = I_p = 1.0, k_d = 0.4$

Light-Source Attenuation

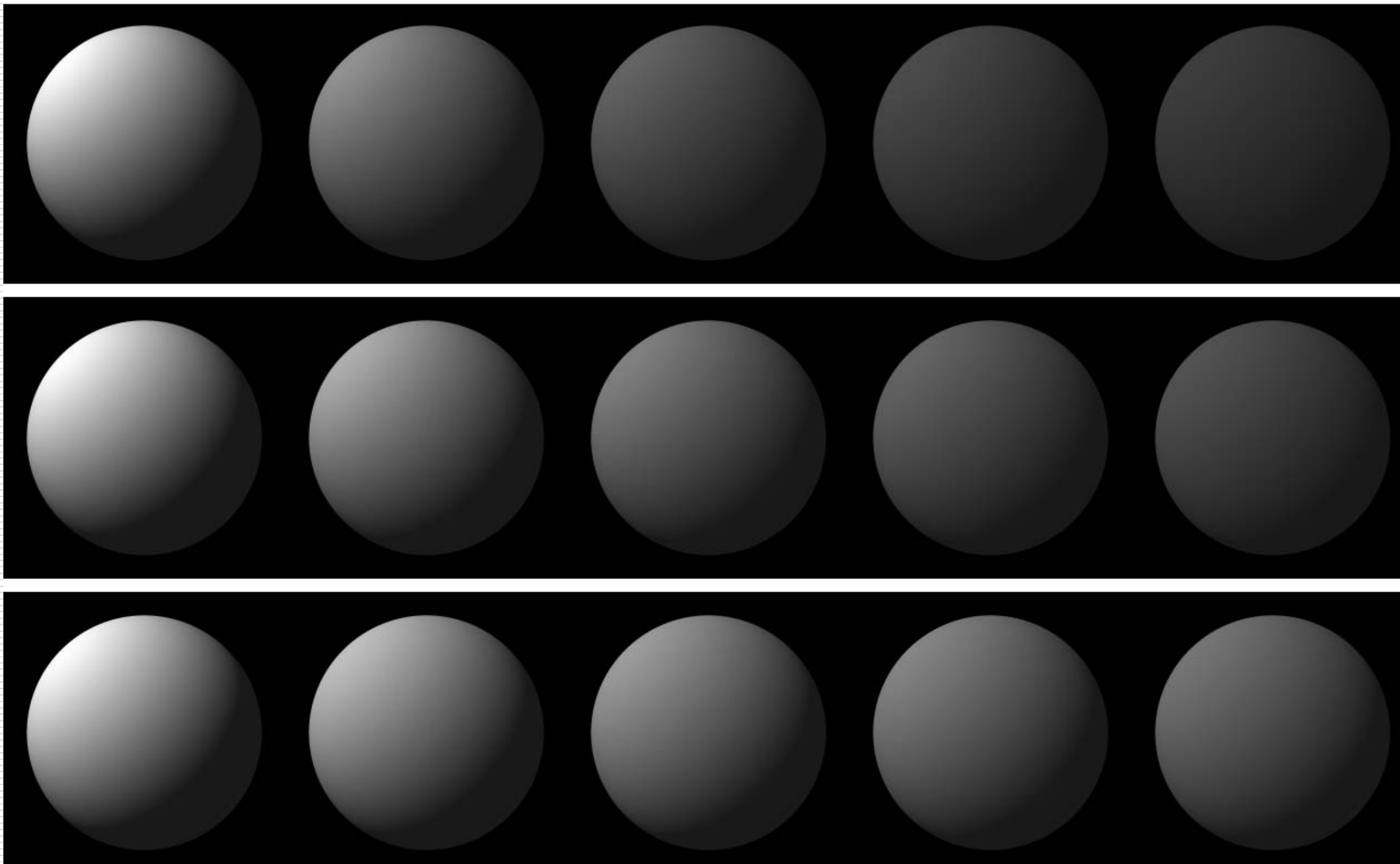
- $I = I_a k_a + f_{att} I_p k_d (\vec{N} \bullet \vec{L})$
  - f_{att} : light-source attenuation factor
  - if the light is a point source

$$f_{att} = \frac{1}{d_L^2}$$

  - where d_L is the distance the light travels from the point source to the surface

$$f_{att} = \min\left(\frac{1}{c_1 + c_2 d_L + c_3 d_L^2}, 1\right)$$

Examples



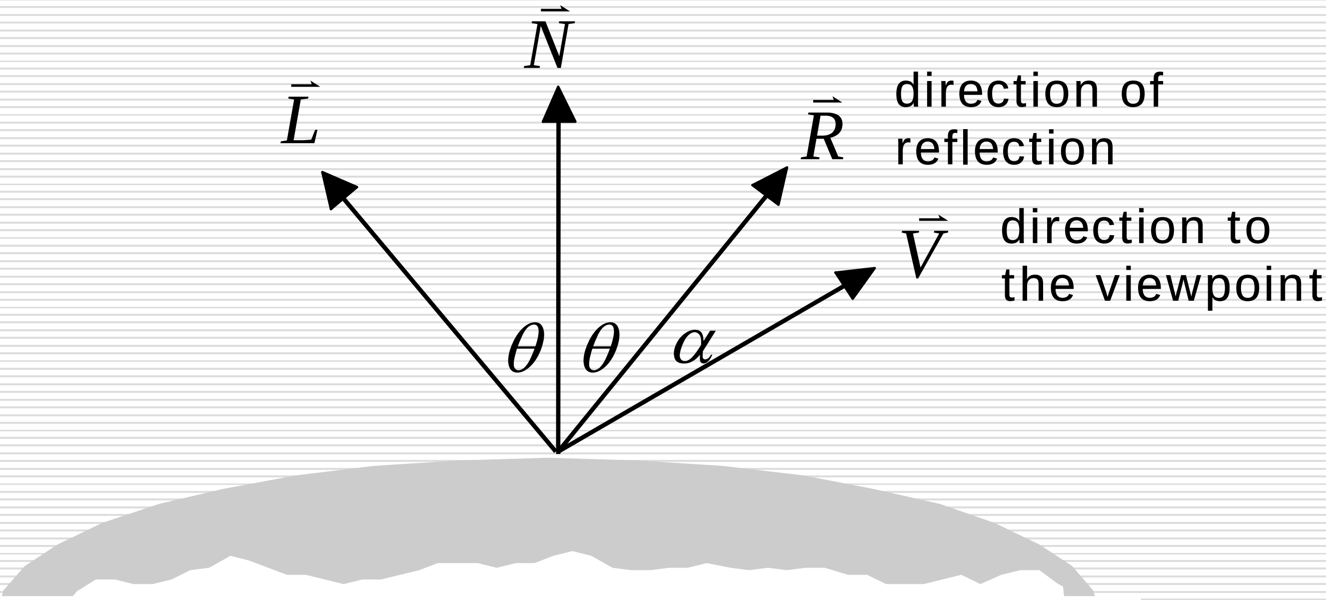
$$\frac{1}{d_L^2}$$

$$\frac{1}{d_L}$$

Colored Lights and Surfaces

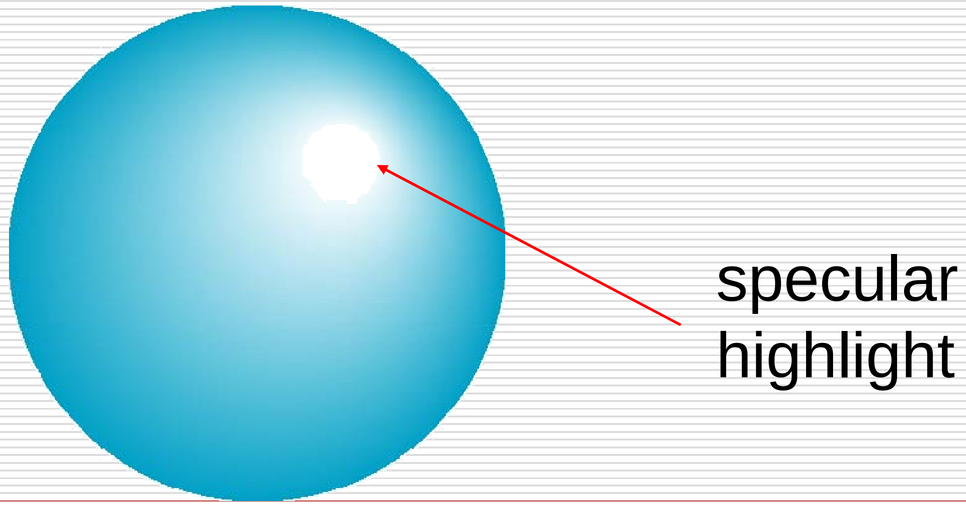
- If an object's **diffuse color** is $O_{\mathrm{d}} = (O_{\mathrm{dR}}, O_{\mathrm{dG}}, O_{\mathrm{dB}})$ then $I = (I_{\mathrm{R}}, I_{\mathrm{G}}, I_{\mathrm{B}})$ where for the red component
  $$I_{\mathrm{R}} = I_{\mathrm{aR}} k_{\mathrm{a}} O_{\mathrm{dR}} + f_{\mathrm{att}} I_{\mathrm{pR}} k_{\mathrm{d}} O_{\mathrm{dR}} (\vec{N} \bullet \vec{L})$$
  however, it should be
  $$I_{\lambda} = I_{\mathrm{a}\lambda} k_{\mathrm{a}} O_{\mathrm{d}\lambda} + f_{\mathrm{att}} I_{\mathrm{p}\lambda} k_{\mathrm{d}} O_{\mathrm{d}\lambda} (\vec{N} \bullet \vec{L})$$
  where λ is the **wavelength**

Specular Reflection



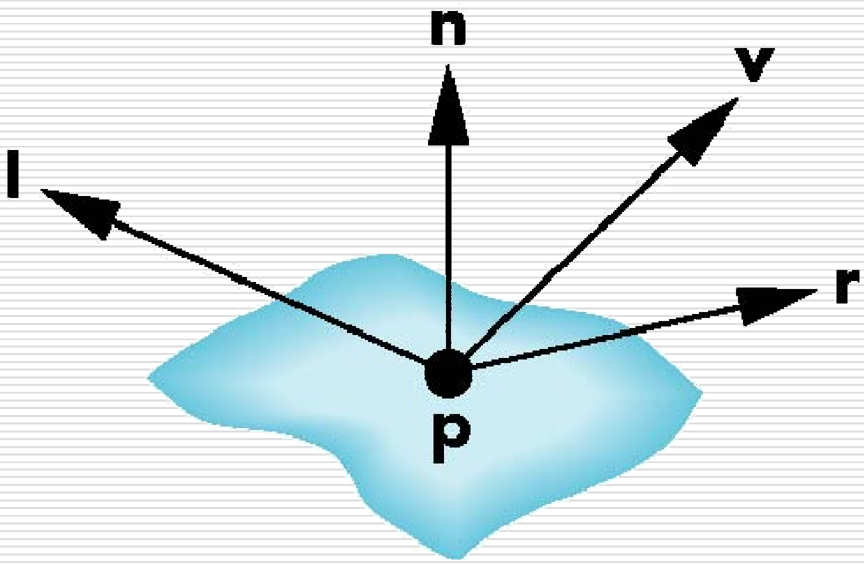
Specular Surfaces

- Most surfaces are neither ideal diffusers nor perfectly specular (ideal refectors)
- Smooth surfaces show specular highlights due to incoming light being reflected in directions concentrated close to the direction of a perfect reflection



The Phong Illumination Model

- A simple model that can be computed rapidly
- Has three components
  - Diffuse
  - Specular
  - Ambient
- Uses four vectors
  - To source
  - To viewer
  - Normal
  - Perfect reflector



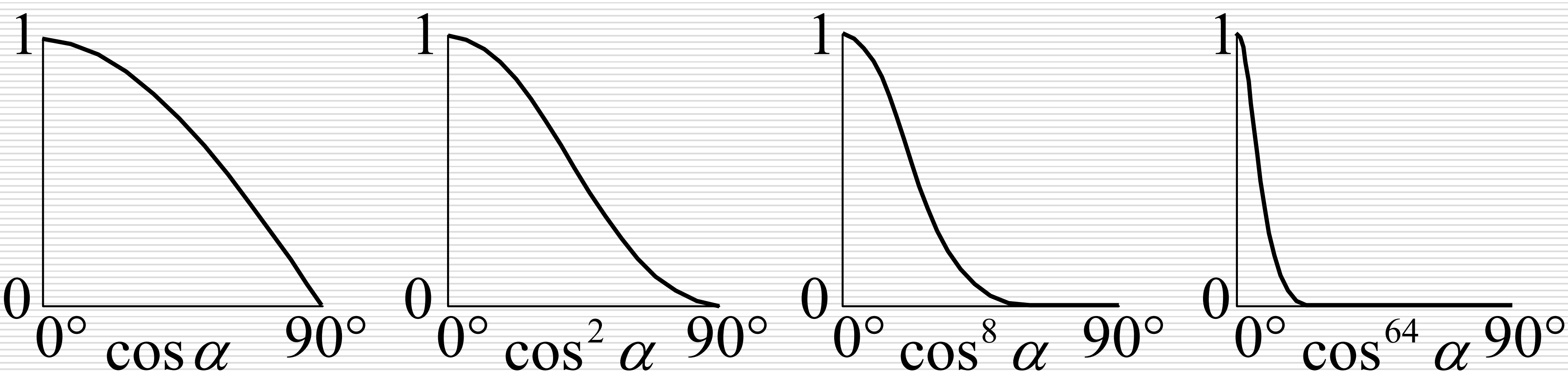
The Phong Illumination Model

- $I_\lambda = I_{a\lambda} k_a O_{d\lambda} + f_{att} I_{p\lambda} [k_d O_{d\lambda} \cos\theta + W(\theta) \cos^n \alpha]$
  - $W(\theta) = k_s$: specular-reflection coefficient: 0~1
- so, the Eq. can be rewritten as
  $I_\lambda = I_{a\lambda} k_a O_{d\lambda} + f_{att} I_{p\lambda} [k_d O_{d\lambda} (\vec{N} \bullet \vec{L}) + k_s (\vec{R} \bullet \vec{V})^n]$
- consider the object's **specular color**

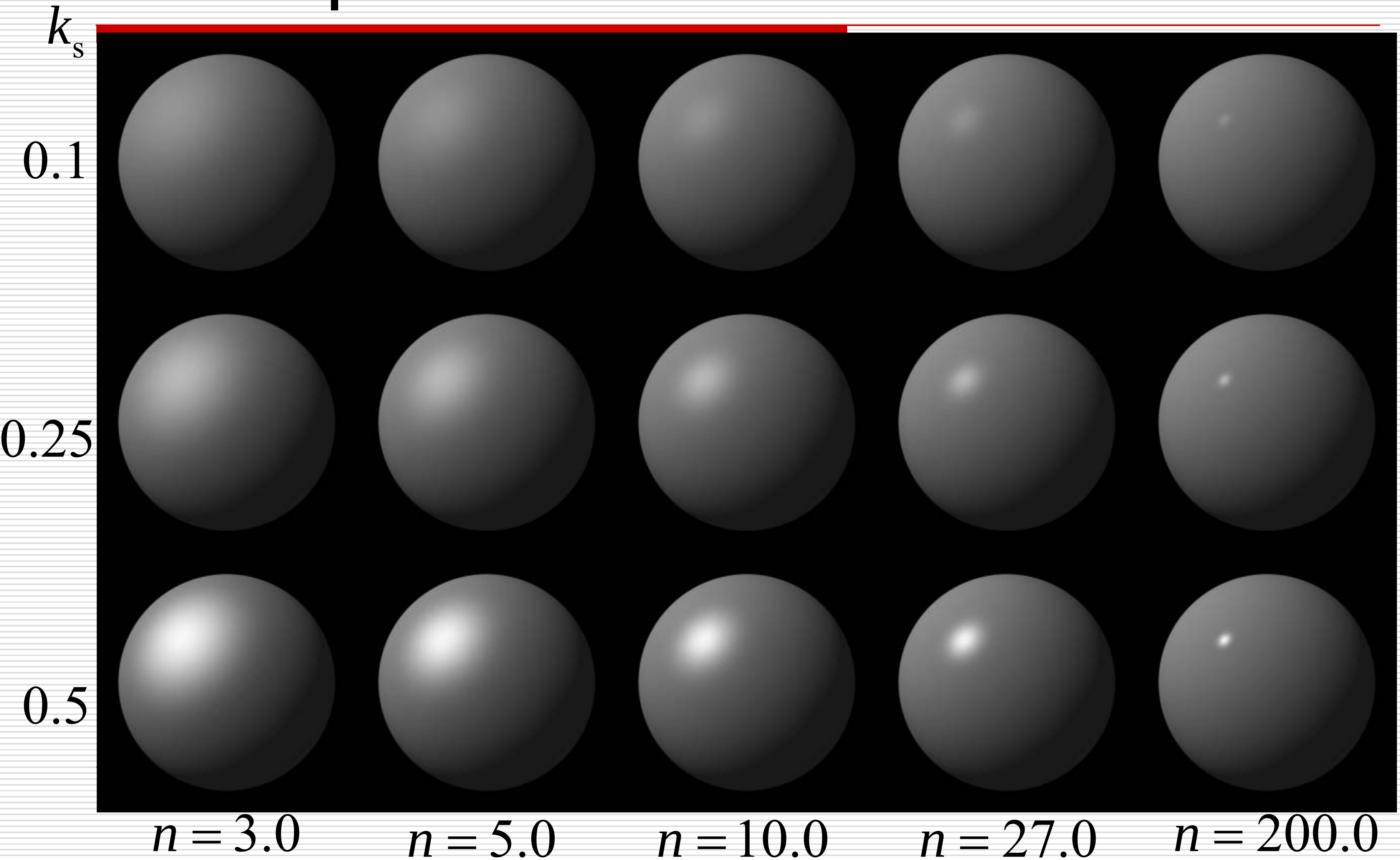
$$I_\lambda = I_{a\lambda} k_a O_{d\lambda} + f_{att} I_{p\lambda} [k_d O_{d\lambda} (\vec{N} \bullet \vec{L}) + k_s O_{s\lambda} (\vec{R} \bullet \vec{V})^n]$$

  - $O_{s\lambda}$: specular color

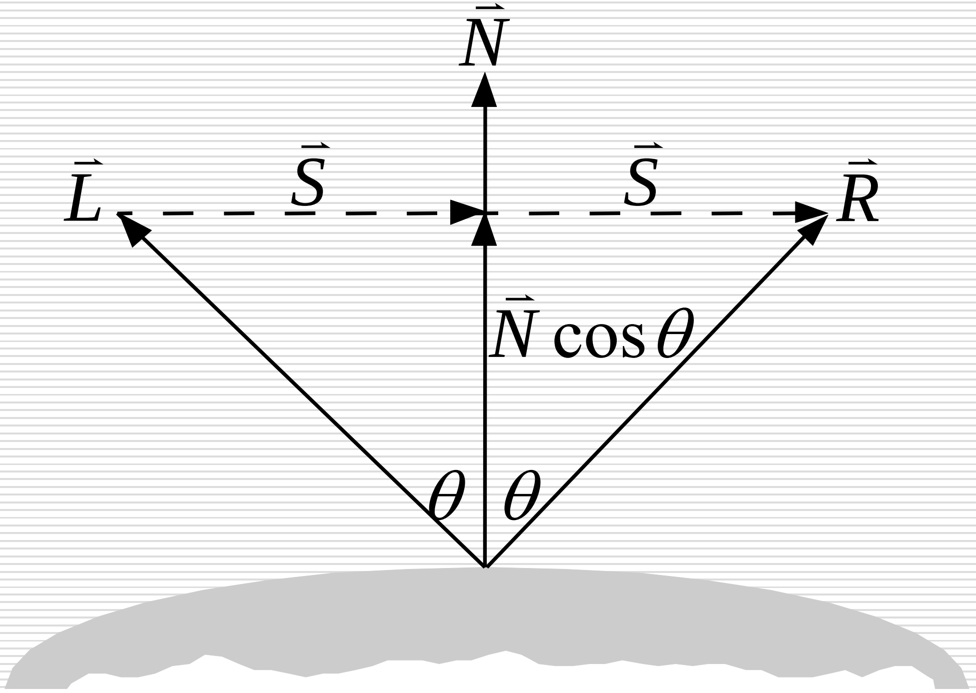
The Phong Illumination Model



Examples

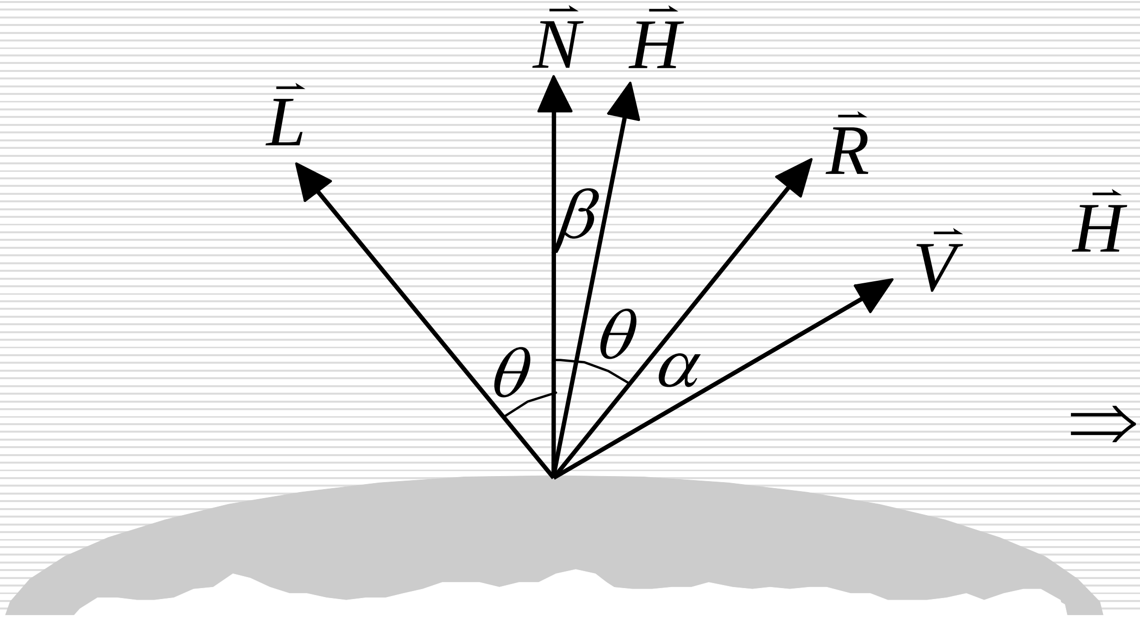


Calculating the Reflection Vector



$$\vec{R} = \vec{N}\cos\theta + \vec{S}$$
$$= \vec{N}\cos\theta + \vec{N}\cos\theta - \vec{L}$$
$$= 2\vec{N}\cos\theta - \vec{L}$$
$$= 2\vec{N}(\vec{N}\bullet\vec{L}) - \vec{L}$$

The Halfway Vector



$$\vec{H} = \frac{\vec{L} + \vec{V}}{\left|\vec{L} + \vec{V}\right|}$$

$$\Rightarrow \cos\alpha \approx \vec{N} \bullet \vec{H}$$

Multiple Light Sources

- If there are m light sources, then

$$I_\lambda = I_{a\lambda} k_a O_{d\lambda} + \sum_{1 \le i \le m} f_{att_i} I_{p\lambda_i} [k_d O_{d\lambda} (\vec{N} \bullet \vec{L}_i) + k_s O_{s\lambda} (\vec{R}_i \bullet \vec{V})^n]$$

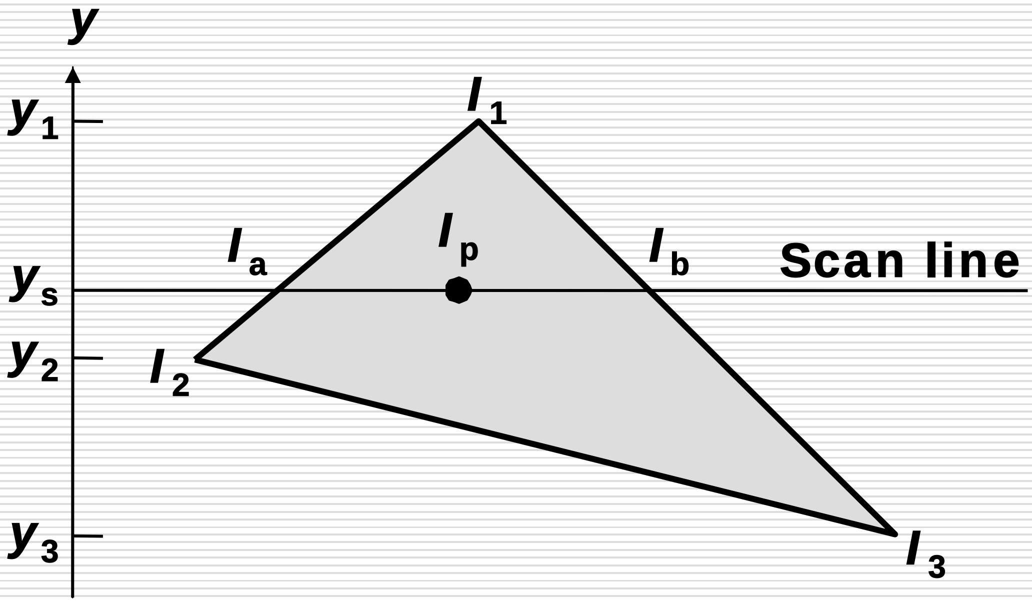
Shading Models for Polygons

- Constant Shading
  - Faceted Shading
  - Flat Shading
- Gouraud Shading
  - Intensity Interpolation Shading
  - Color Interpolation Shading
- Phong Shading
  - Normal-Vector Interpolation Shading

Constant Shading

- Assumptions
  - The light source is at infinity
  - The viewer is at infinity
  - The polygon represents the actual surface being modeled and is not an approximation to a curved surface

Gouraud Shading

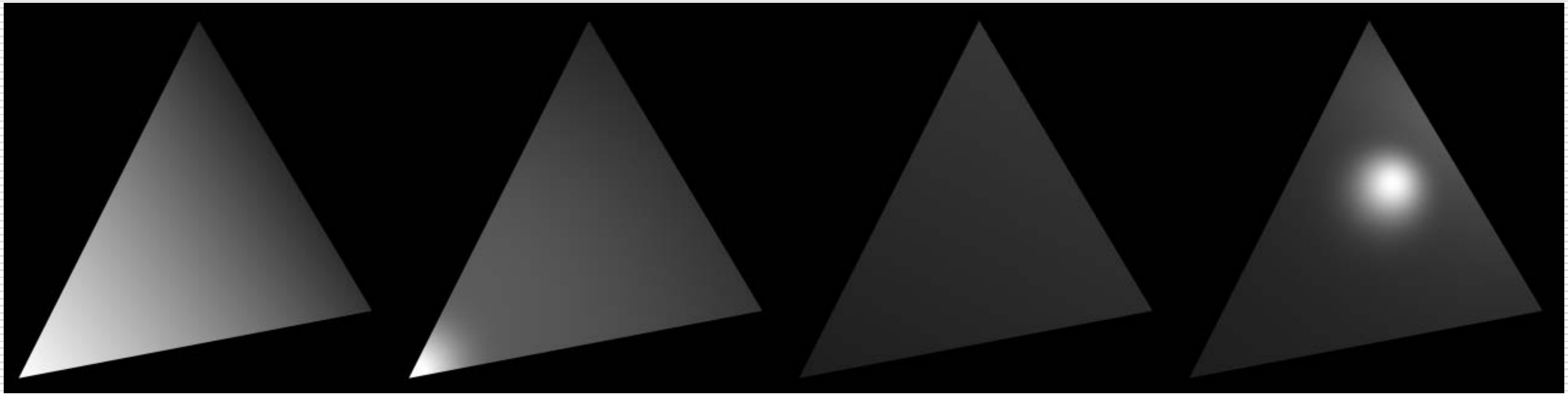


$$I_a = I_1 - (I_1 - I_2)\frac{y_1 - y_s}{y_1 - y_2}$$

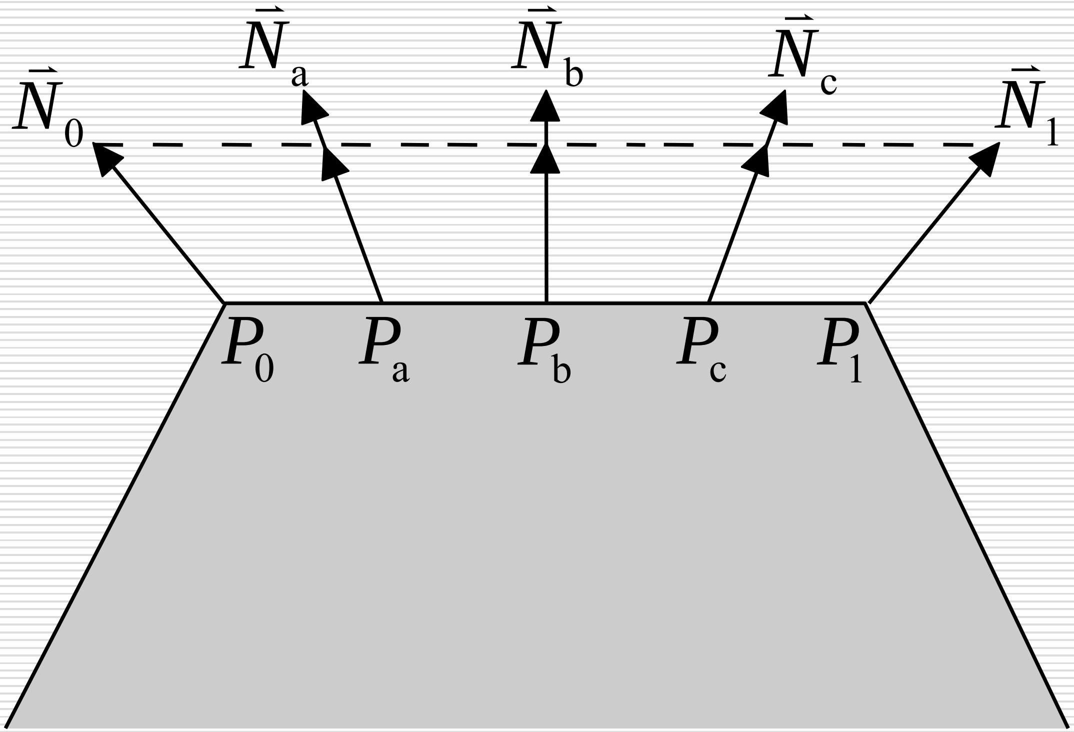
$$I_b = I_1 - (I_1 - I_3)\frac{y_1 - y_s}{y_1 - y_3}$$

$$I_p = I_b - (I_b - I_a)\frac{x_b - x_p}{x_b - x_a}$$

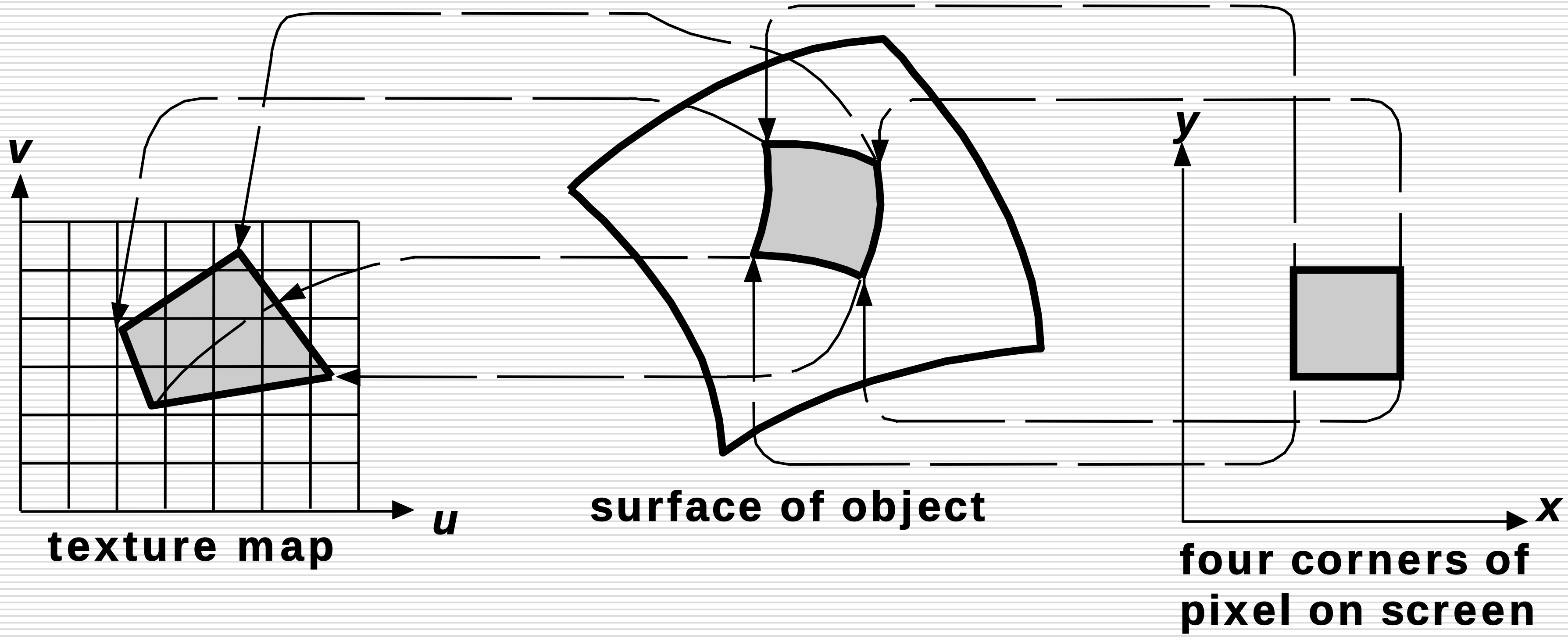
Gouraud v.s. Phong Shading



Phong Shading



Texture Mapping = Pattern Mapping

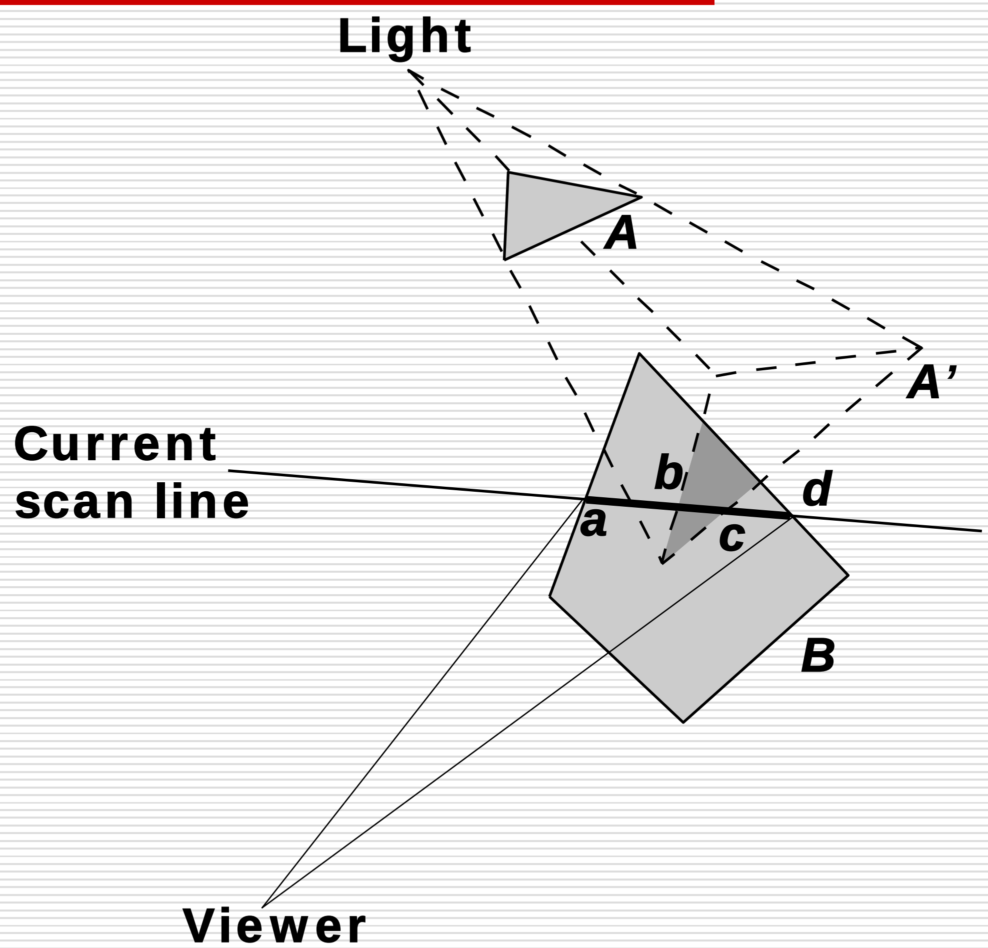


Bump Mapping & Displacement Mapping

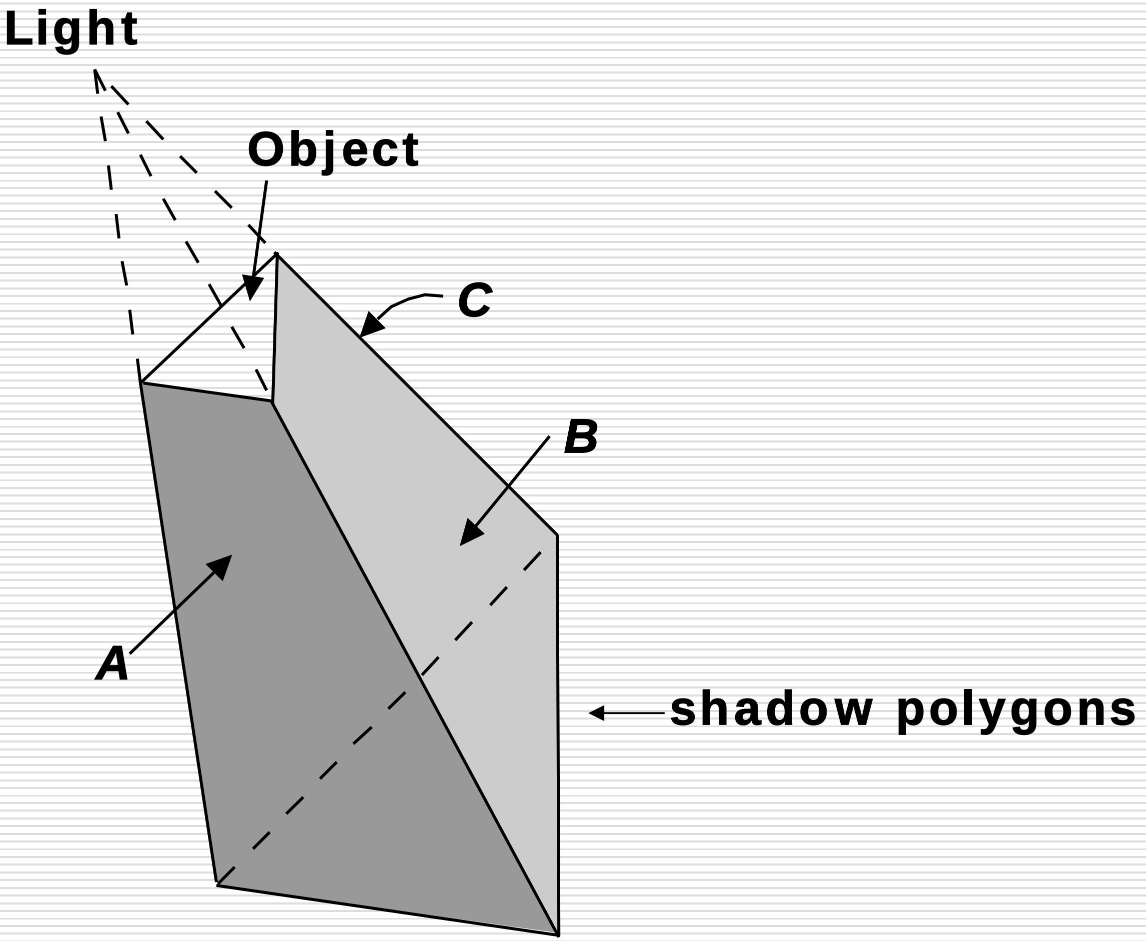
Shadows

- $I_\lambda = I_{a\lambda} k_a O_{d\lambda} + \sum_{1 \le i \le m} S_i f_{\mathrm{att}_i} I_{p\lambda_i} [k_d O_{d\lambda} (\vec{N} \bullet \vec{L}_i) + k_s O_{s\lambda} (\vec{R}_i \bullet \vec{V})^n]$
  - $S_i = \begin{cases} 0, \text{if light } i \text{ is blocked at this point} \\ 1, \text{if light } i \text{ is not blocked at this point} \end{cases}$

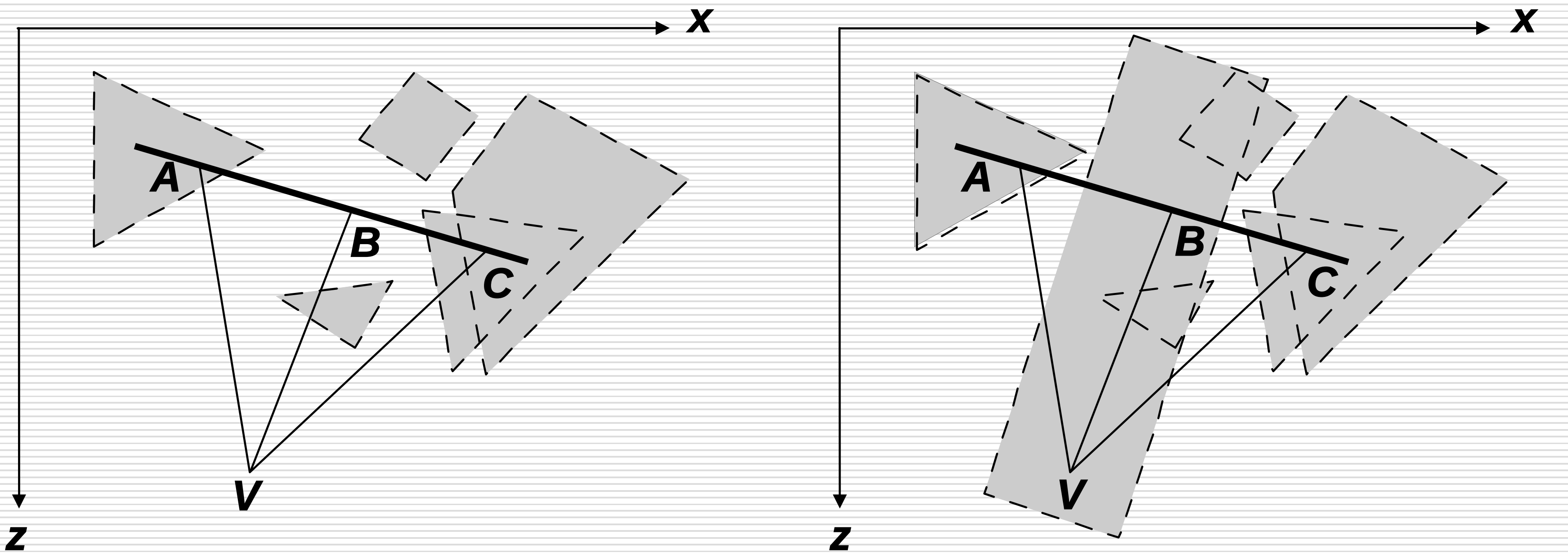
Scan-Line Generation of Shadows



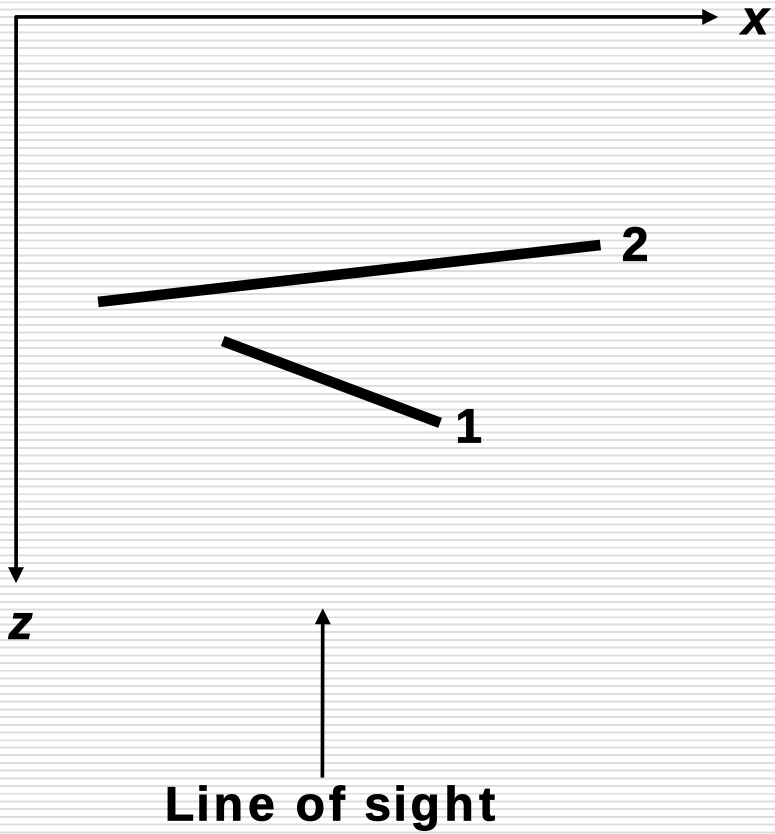
Shadow Volumes



Shadow Volumes



Transparency

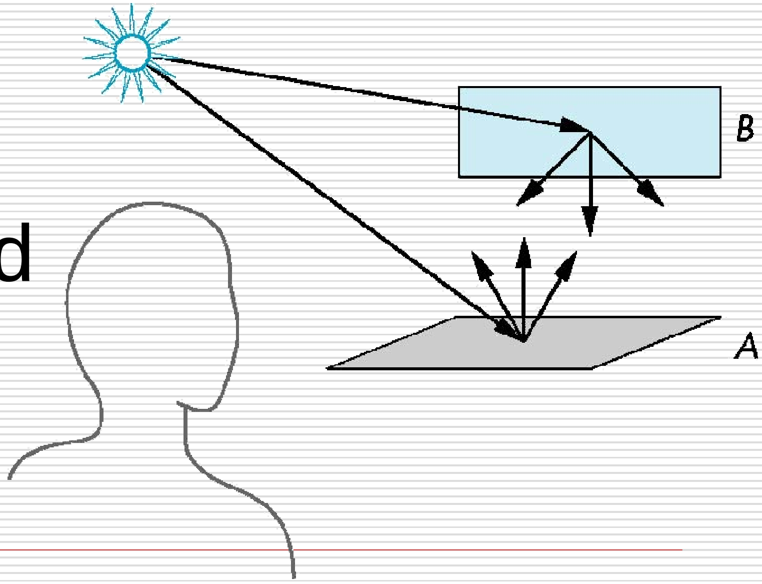


- interpolated transparency
- $I_\lambda = (1 - k_{t1}) I_{\lambda 1} + k_{t1} I_{\lambda 2}$
  - k_{t1}: transparency: 0 ~ 1
- filtered transparency
- $I_\lambda = I_{\lambda 1} + k_{t1} O_{t\lambda} I_{\lambda 2}$
  - $O_{t\lambda}$: transparency color

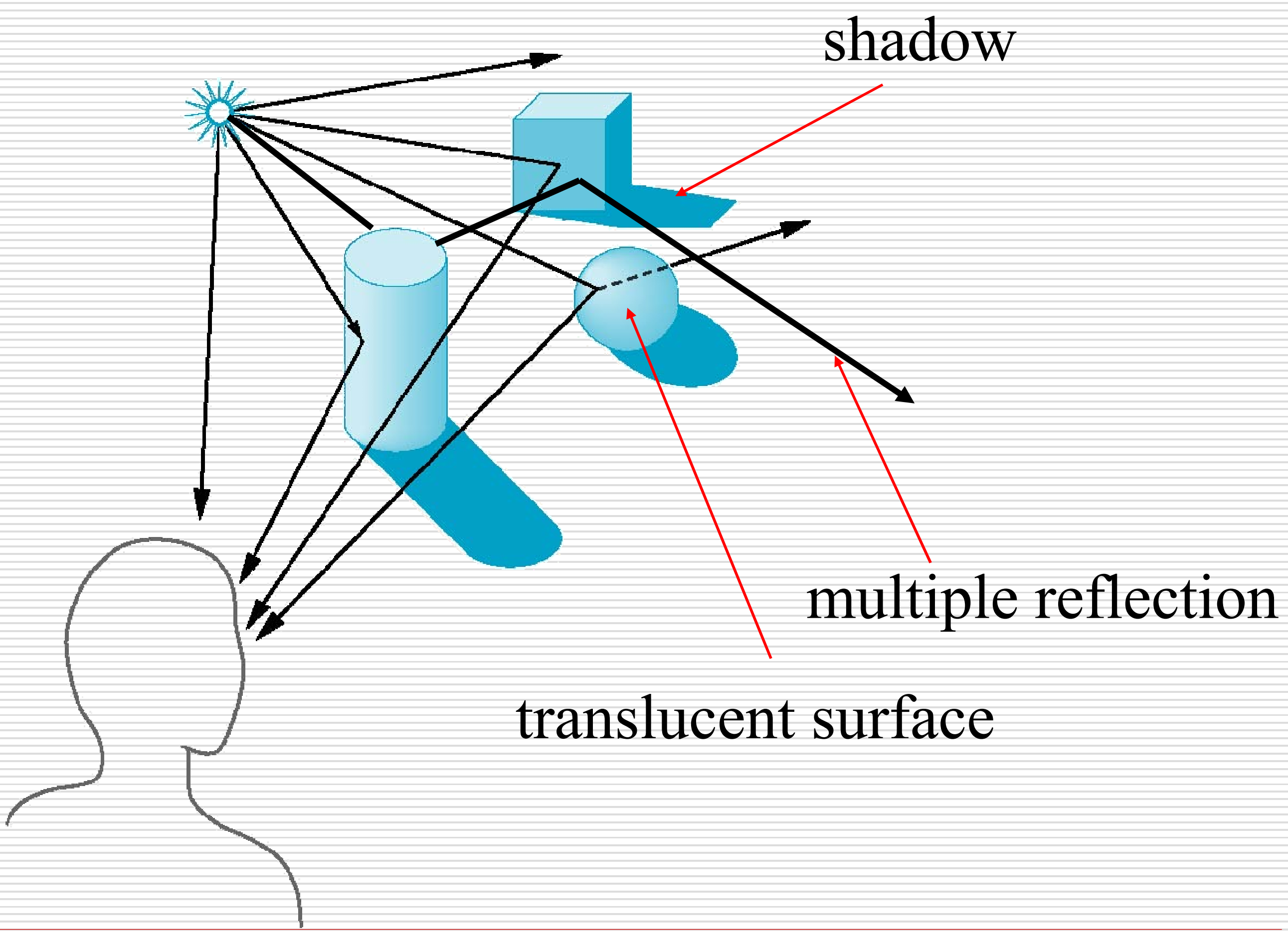
Scattering

- Light strikes A
  - Some scattered
  - Some absorbed
- Some of scattered light strikes B
  - Some scattered
  - Some absorbed
- Some of this scattered light strikes A

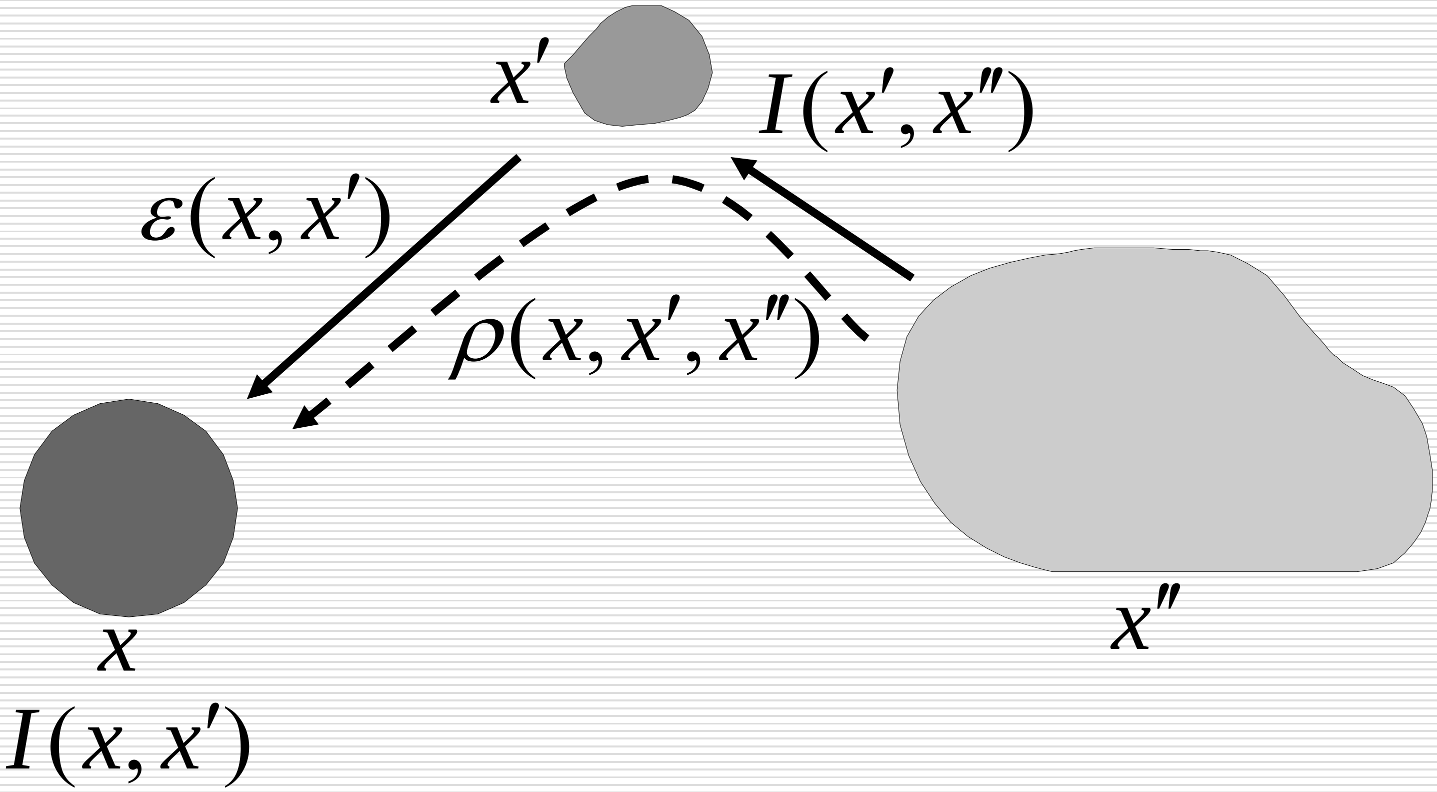
  and so on



Global Effects



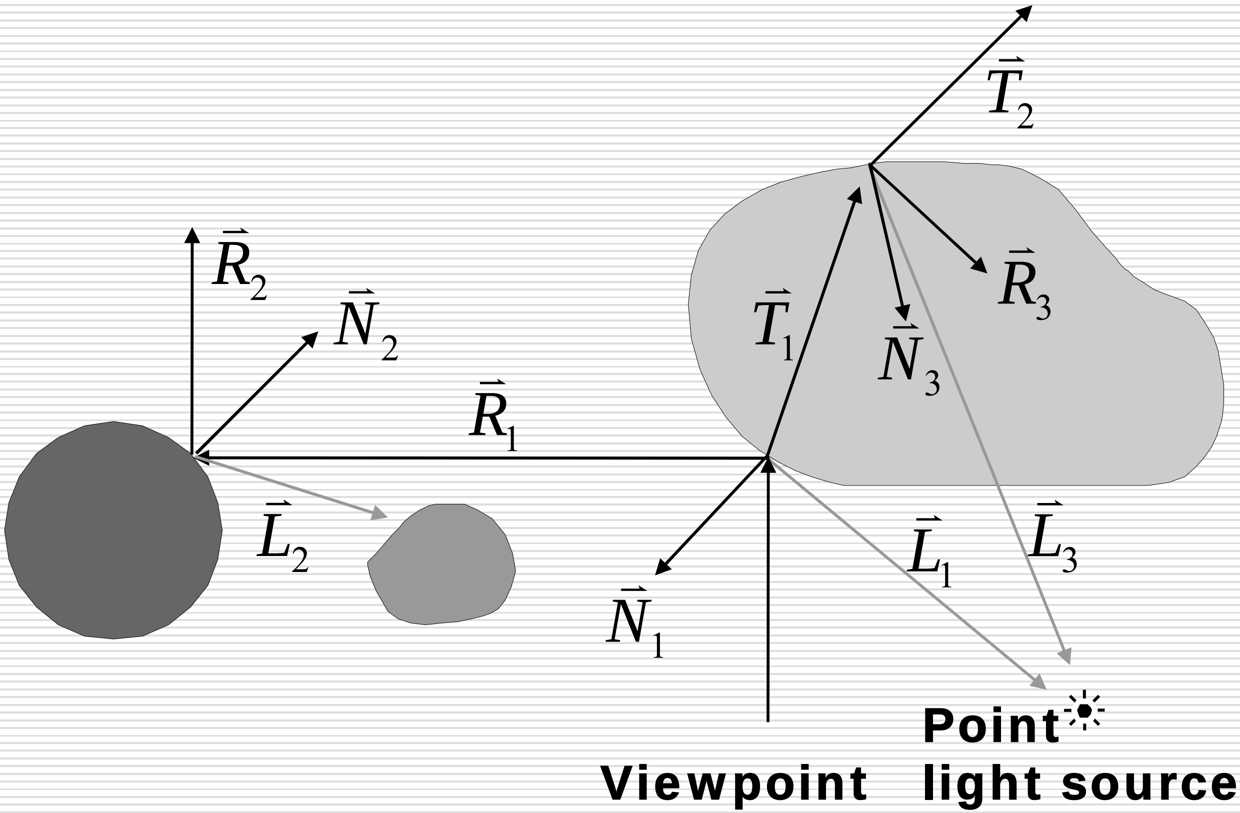
Global Illumination



The Rendering Equation

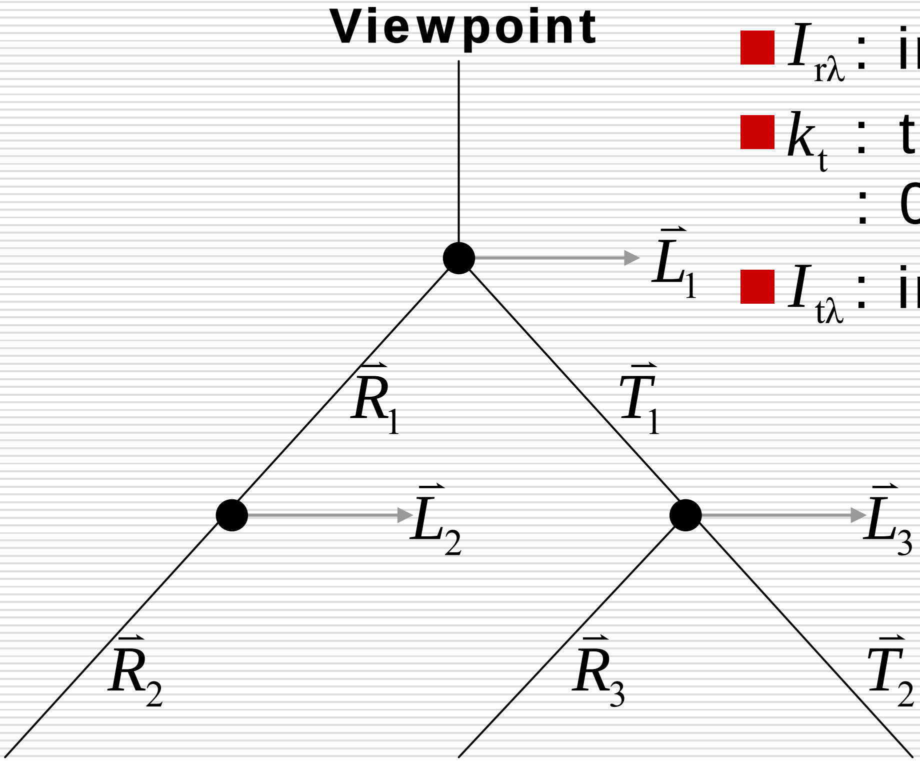
- $I(x,x') = g(x,x')\left[\varepsilon(x,x') + \int_S \rho(x,x',x'')I(x',x'')dx''\right]$
  - $I(x,x')$: intensity passing from x' to x
  - $\varepsilon(x,x')$: emitted light intensity from x' to x
  - $\rho(x,x',x'')$: intensity of light reflected from x'' to x from the surface at x'
  - $g(x,x') = \begin{cases} 0, & \text{if } x' \text{ is invisible from } x \\ 1/r^2, & \text{if } x' \text{ is visible from } x \end{cases}$
  - r : the distance between x' and x
  - S : all surfaces

Recursive Ray Tracing



$\vec{N}_i$ **surface normal**
$\vec{R}_i$ **reflected ray**
$\vec{L}_i$ **shadow ray**
$\vec{T}_i$ **transmitted ray**

The Ray Tree



- $I_{r\lambda}$: intensity of reflected ray
- k_t : transmission coefficient
  : 0 ~ 1
- $I_{t\lambda}$: intensity of transmitted ray

$$I_\lambda = I_{a\lambda} k_a O_{d\lambda} + \sum_{1 \le i \le m} S_i f_{att_i} I_{p\lambda_i} [k_d O_{d\lambda} (\vec{N} \bullet \vec{L}_i) + k_s (\vec{N} \bullet \vec{H}_i)^n] + k_s I_{r\lambda} + k_t I_{t\lambda}$$

The Radiosity Equation

- $$B_i = E_i + \rho_i \sum_{1 \le j \le n} B_j F_{j-i} \frac{A_j}{A_i}$$
  - B_i: radiosity of patch i
  - E_i: rate at which light is emitted from patch i
  - ρ_i: reflectivity of patch i
  - F_{j-i} : form factor (configuration factor)
  - A_i: area of patch i
- since $A_i F_{i-j} = A_j F_{j-i}$
- thus $$B_i = E_i + \rho_i \sum_{1 \le j \le n} B_j F_{i-j}$$

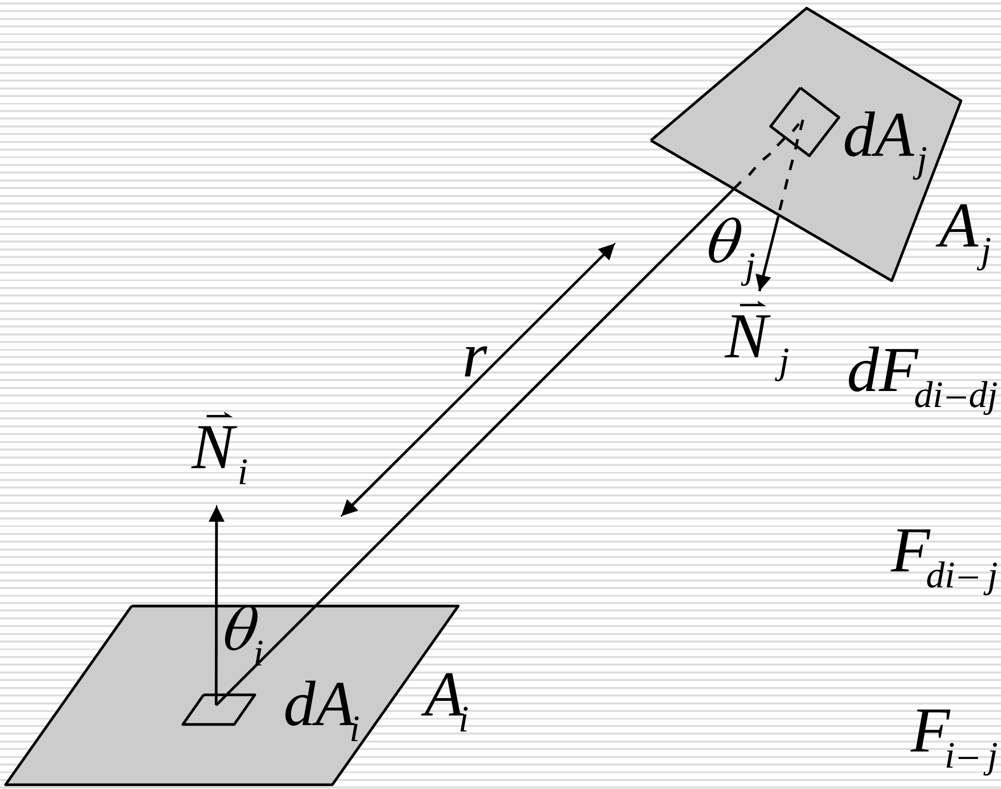
The Radiosity Equation

- rearranging terms $B_i - \rho_i \sum_{1 \le j \le n} B_j F_{i-j} = E_i$
- therefore

$$\begin{bmatrix} 1-\rho_1 F_{1-1} & -\rho_1 F_{1-2} & \cdots & -\rho_1 F_{1-n} \\ -\rho_2 F_{2-1} & 1-\rho_2 F_{2-2} & \cdots & -\rho_2 F_{2-n} \\ \vdots & \vdots & \cdots & \vdots \\ -\rho_n F_{n-1} & -\rho_n F_{n-2} & \cdots & 1-\rho_n F_{n-n} \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_n \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_n \end{bmatrix}$$

- progressive refinement

Computing Form Factors



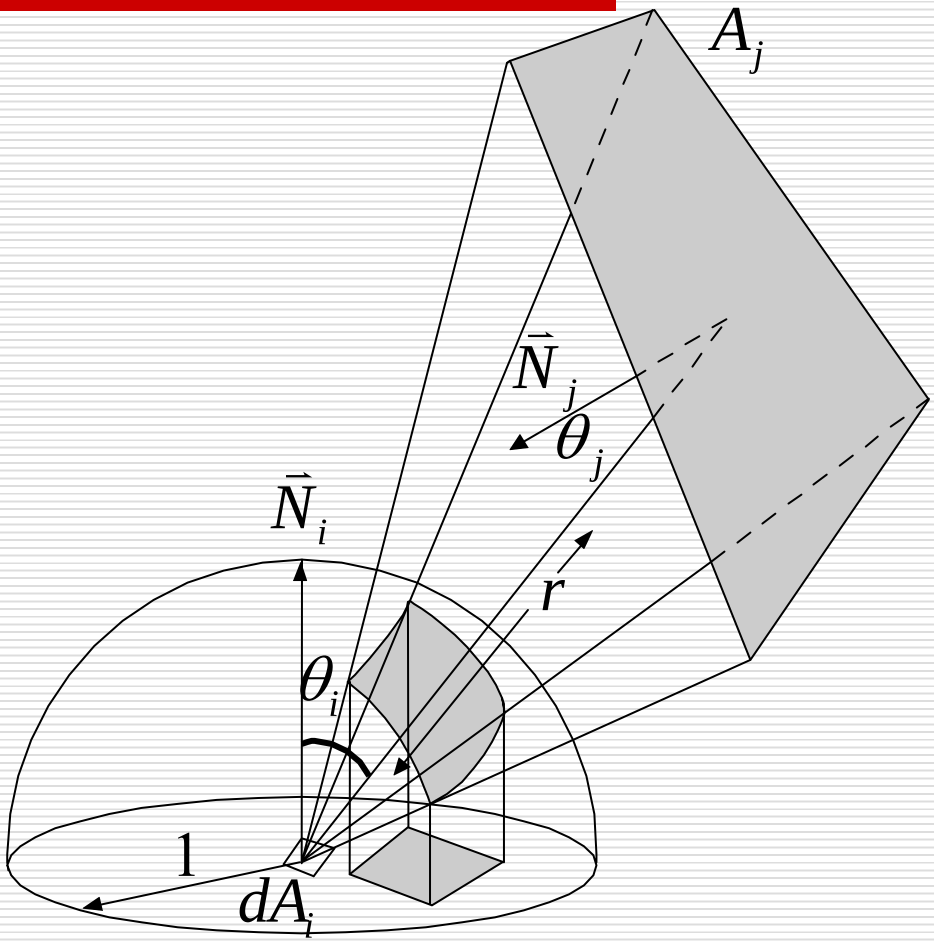
visible or invisible
1 **0**

$$dF_{di-dj} = \frac{\cos\theta_i \cos\theta_j}{\pi r^2} H_{ij} dA_j$$

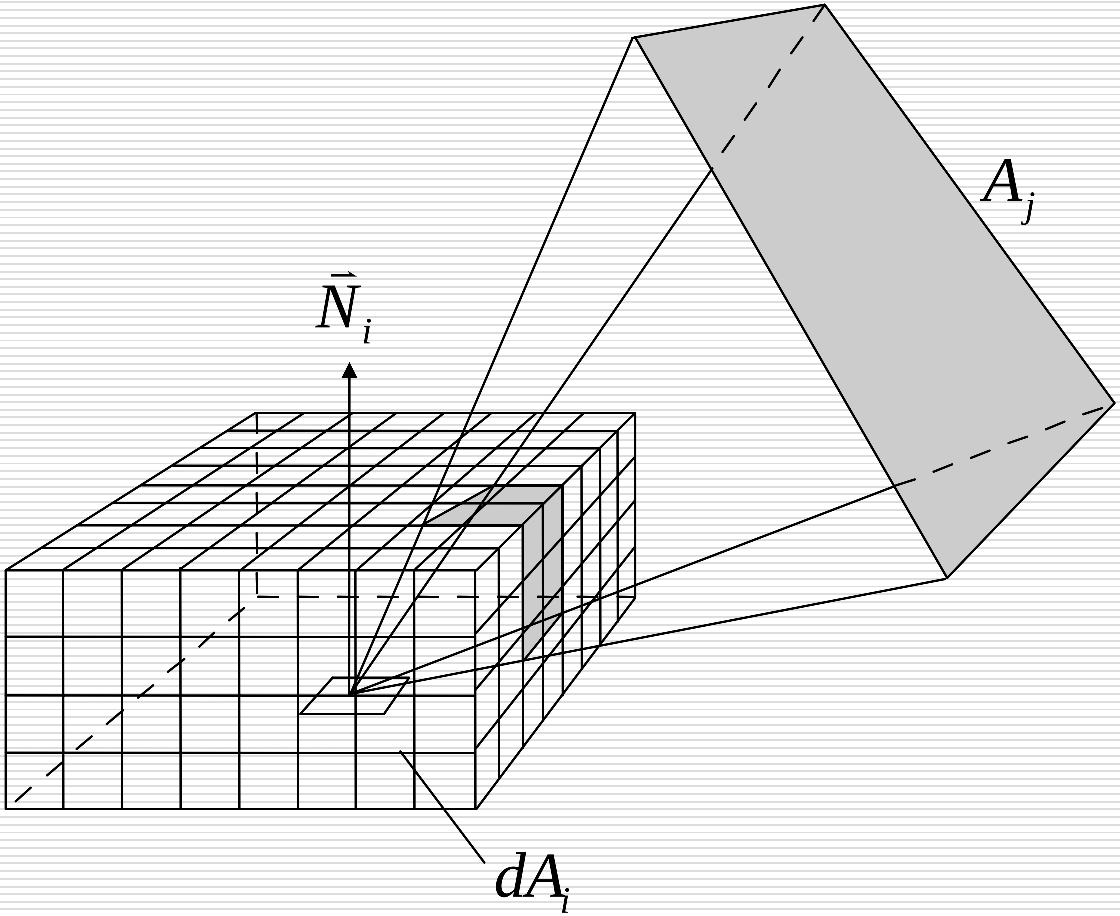
$$F_{di-j} = \int_{A_j} \frac{\cos\theta_i \cos\theta_j}{\pi r^2} H_{ij} dA_j$$

$$F_{i-j} = \frac{1}{A_i} \int_{A_i} \int_{A_j} \frac{\cos\theta_i \cos\theta_j}{\pi r^2} H_{ij} dA_j dA_i$$

Hemisphere



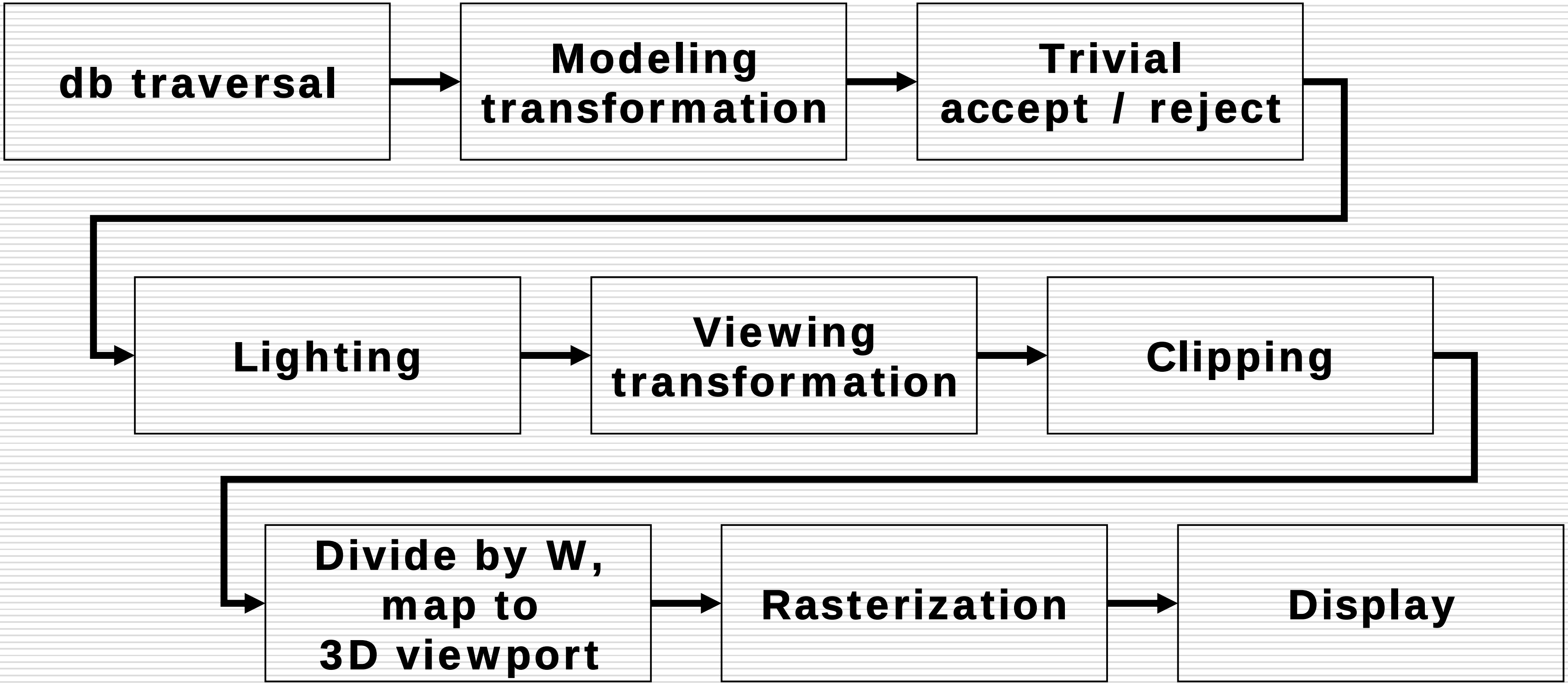
Hemicube



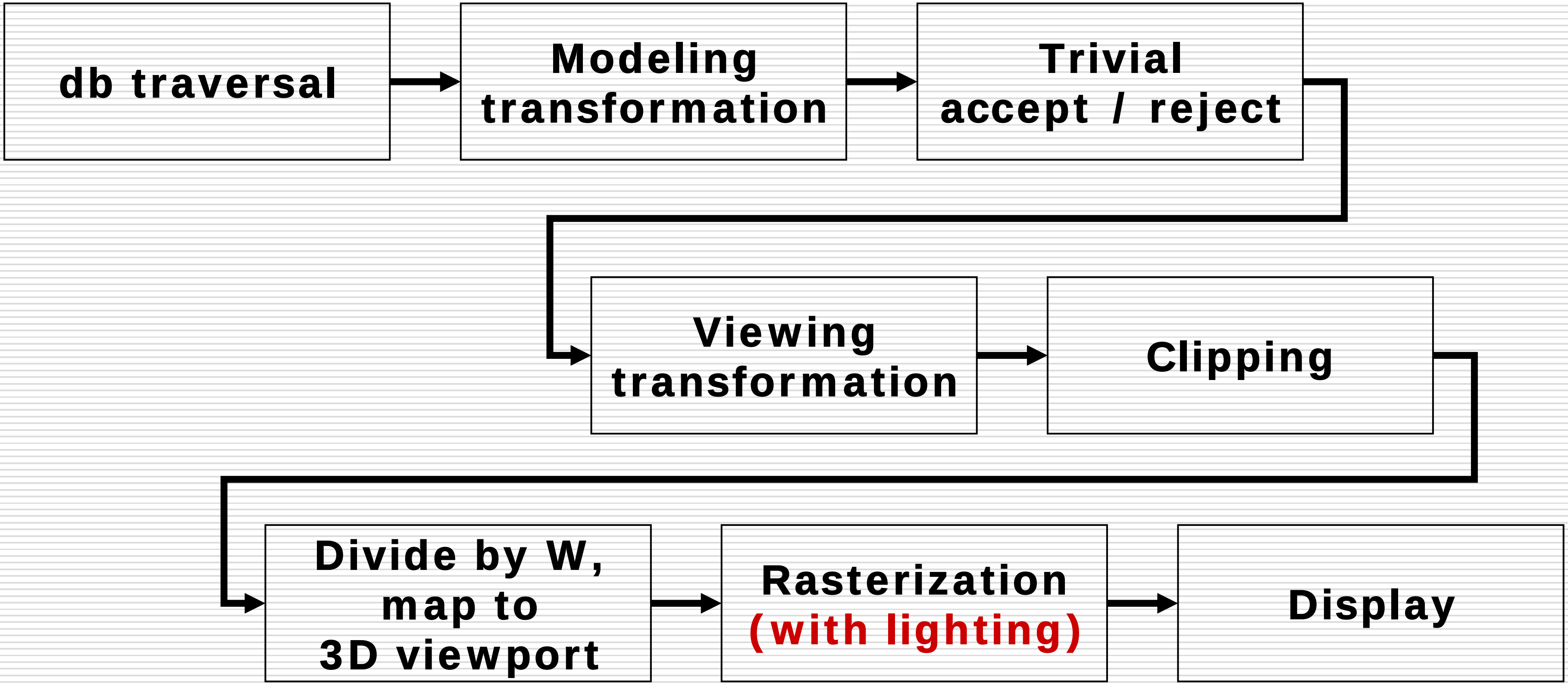
The Rendering Pipeline

- Local Illumination Pipelines
  - z-buffer and Gouraud shading
  - z-buffer and Phong shading
  - list-priority algorithm and Phong shading
- Global Illumination Pipelines
  - radiosity
  - ray tracing

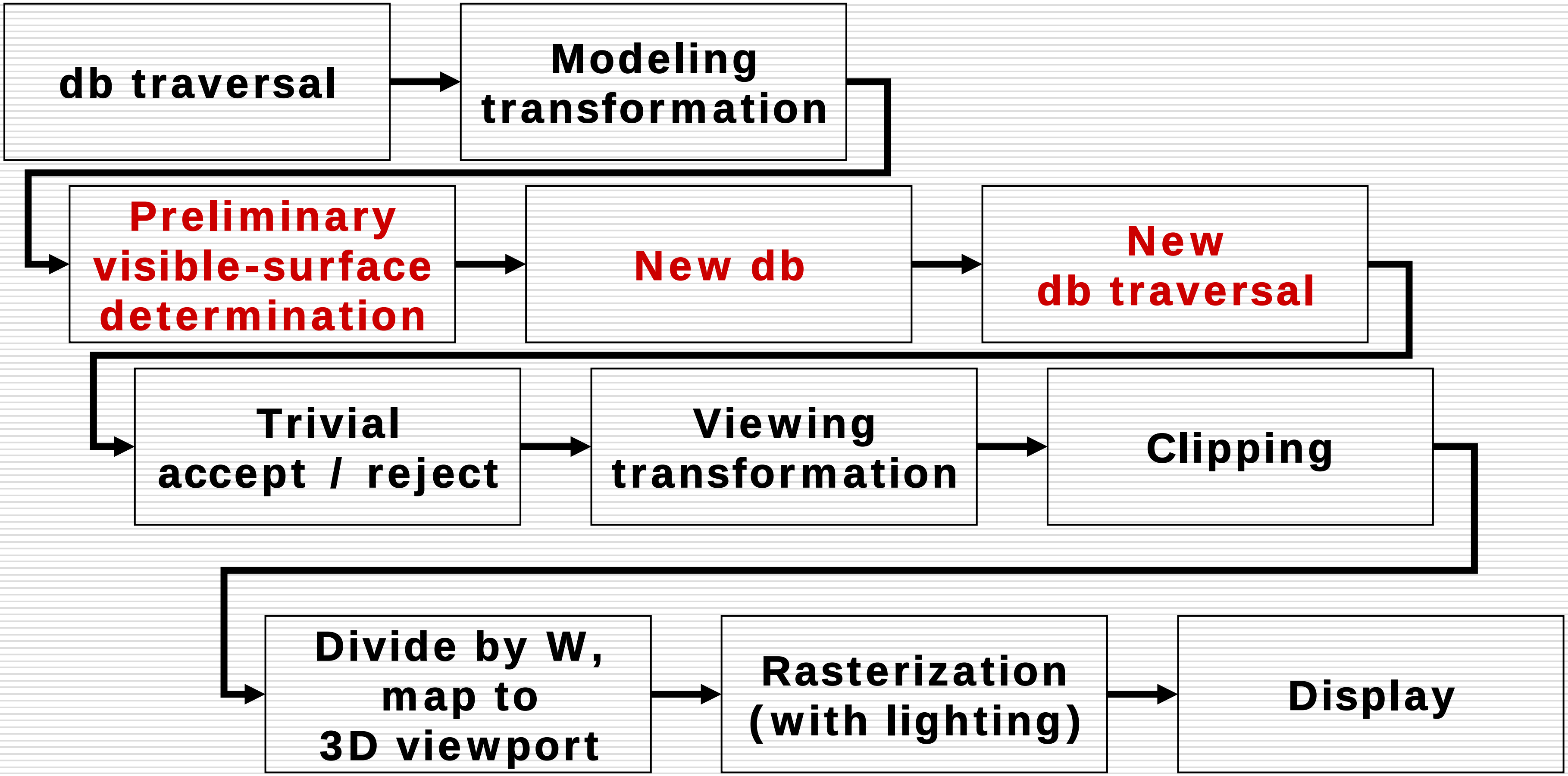
Rendering Pipeline for z-buffer & Gouraud shading



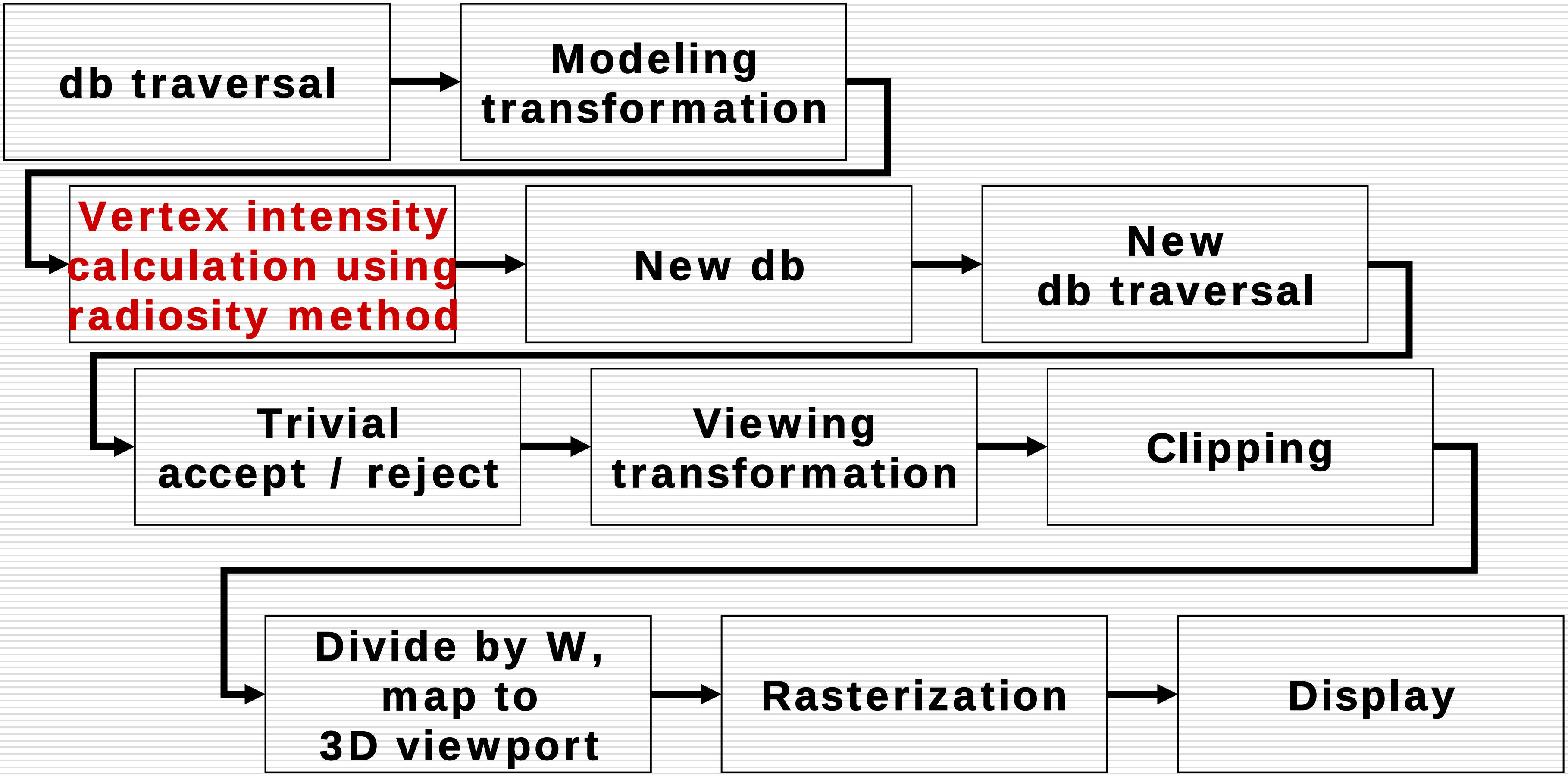
Rendering Pipeline for z-buffer & Phong shading



Rendering Pipeline for list-priority algorithm & Phong shading



Rendering Pipeline for radiosity & Gouraud shading



Rendering Pipeline for ray tracing