Computer Graphics

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(modified from Bing-Yu Chen’s slides)

Introduction to OpenGL

- General OpenGL Introduction
- An Example OpenGL Program
- Drawing with OpenGL
- Transformations
- Animation and Depth Buffering
- Lighting
- Evaluation and NURBS
- Texture Mapping
- Advanced OpenGL Topics
- Imaging

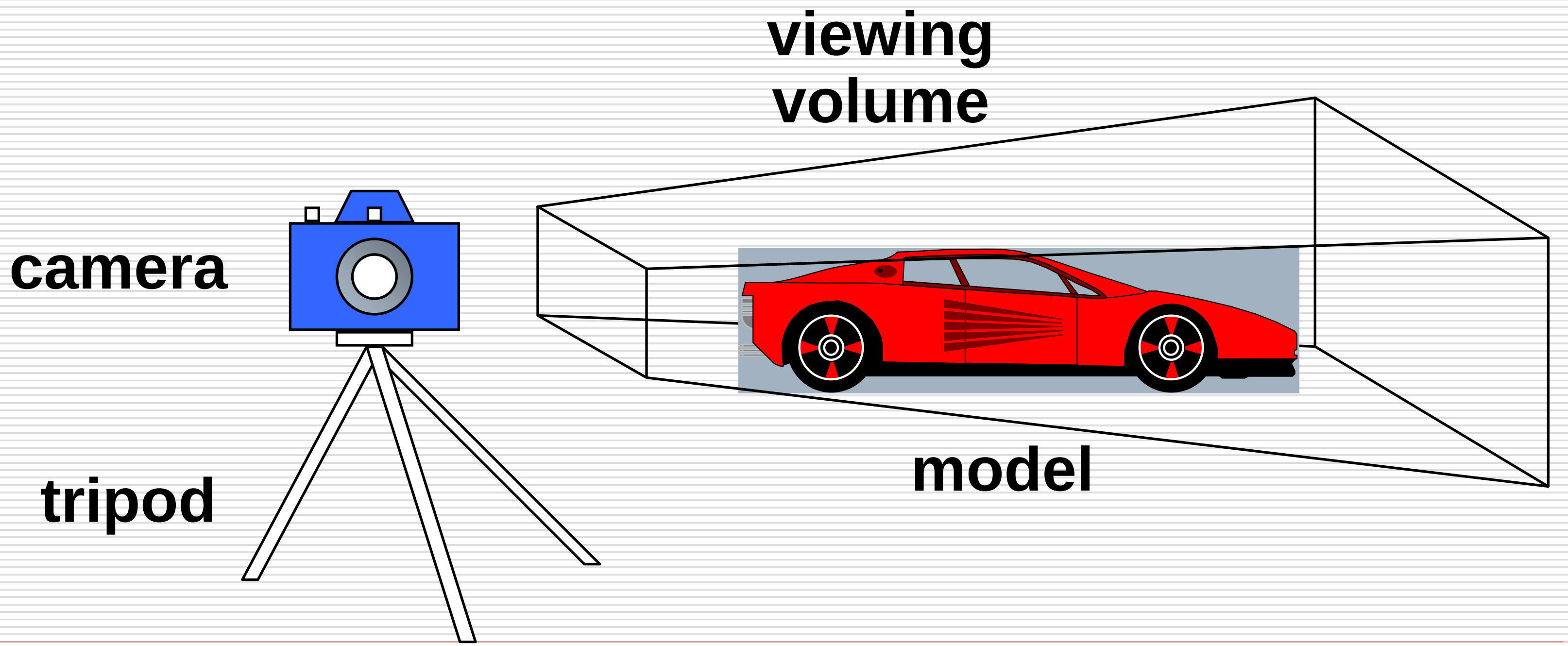
modified from
Dave Shreiner, Ed Angel, and Vicki Shreiner.
An Interactive Introduction to OpenGL Programming.
ACM SIGGRAPH 2001 Conference Course Notes #54.
& *ACM SIGGRAPH 2004 Conference Course Notes #29.*

Transformations in OpenGL

- Modeling
- Viewing
  - orient camera
  - projection
- Animation
- Map to screen

Camera Analogy

- 3D is just like taking a photograph (lots of photographs!)



Camera Analogy & Transformations

- Projection transformations
  - adjust the lens of the camera
- Viewing transformations
  - tripod–define position and orientation of the viewing volume in the world
- Modeling transformations
  - moving the model
- Viewport transformations
  - enlarge or reduce the physical photograph

Coordinate Systems & Transformations

- Steps in Forming an Image
  - specify geometry (world coordinates)
  - specify camera (camera coordinates)
  - project (window coordinates)
  - map to viewport (screen coordinates)
- Each step uses transformations
- Every transformation is equivalent to a change in coordinate systems (frames)

Affine Transformations

- Want transformations which preserve geometry
  - lines, polygons, quadrics
- Affine = line preserving
  - Rotation, translation, scaling
  - Projection
  - Concatenation (composition)

Homogeneous Coordinates

- each vertex is a column vector

$$\vec{v} = \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}$$

- w is usually 1.0
- all operations are matrix multiplications
- directions (directed line segments) can be represented with w = 0.0

3D Transformations

- A vertex is transformed by 4 x 4 matrices
  - all affine operations are matrix multiplications
  - all matrices are stored column-major in OpenGL
  - matrices are always post-multiplied
  - product of matrix and vector is $\mathbf{M}\vec{v}$

$$\mathbf{M} = \begin{bmatrix} m_0 & m_4 & m_8 & m_{12} \\ m_1 & m_5 & m_9 & m_{13} \\ m_2 & m_6 & m_{10} & m_{14} \\ m_3 & m_7 & m_{11} & m_{15} \end{bmatrix}$$

Specifying Transformations

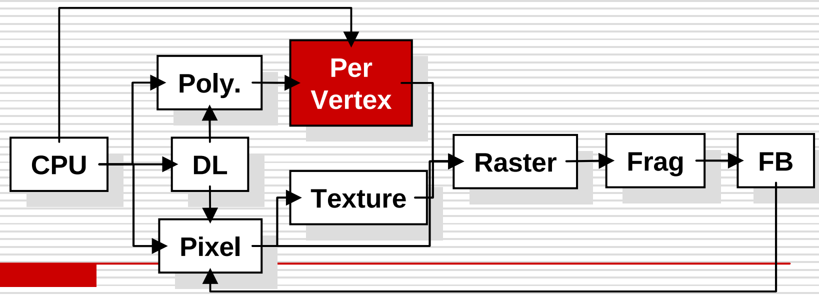
- Programmer has two styles of specifying transformations
  - specify matrices (**glLoadMatrix, glMultMatrix**)
  - specify operation (**glRotate, glOrtho**)

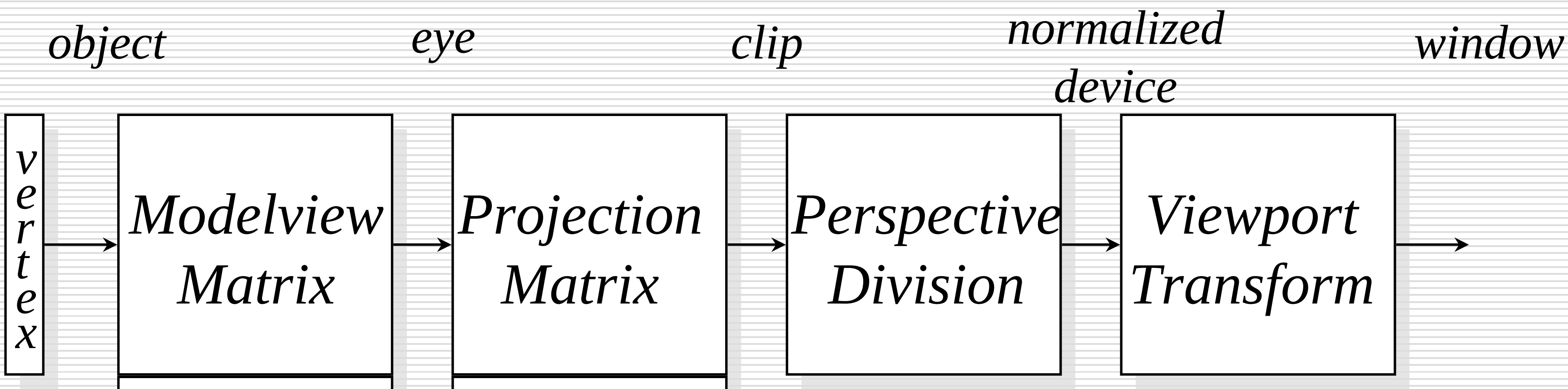
- Programmer does not have to remember the exact matrices
  - check appendix of Red Book (Programming Guide)

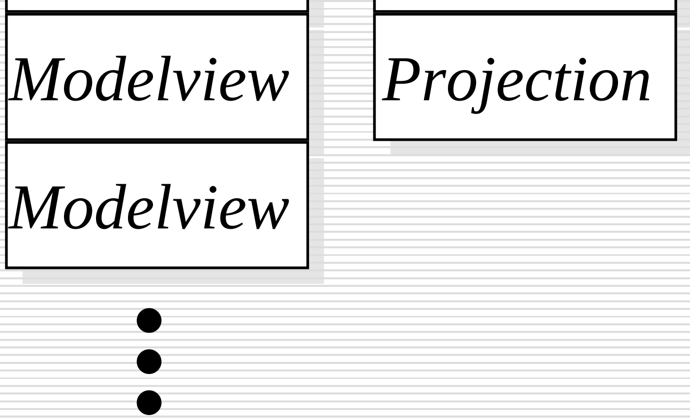
Programming Transformations

- Prior to rendering, view, locate, and orient:
  - eye/camera position
  - 3D geometry
- Manage the matrices
  - including matrix stack
- Combine (composite) transformations

Transformation Pipeline







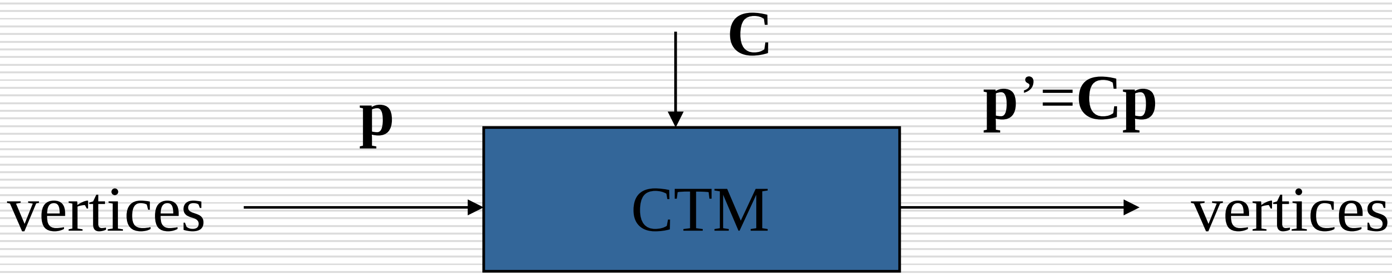
- other calculations here
  - material → color
  - shade model (flat)
  - polygon rendering mode
  - polygon culling
  - clipping

OpenGL Matrices

- In OpenGL matrices are part of the state
- Three types
  - Model-View (**GL_MODEL_VIEW**)
  - Projection (**GL_PROJECTION**)
  - Texture (**GL_TEXTURE**) (ignore for now)
- Single set of functions for manipulation
- Select which to manipulated by
  - **glMatrixMode(GL_MODEL_VIEW);**
  - **glMatrixMode(GL_PROJECTION);**

Current Transformation Matrix (CTM)

- Conceptually there is a 4 x 4 homogeneous coordinate matrix, the *current transformation matrix* (CTM) that is part of the state and is applied to all vertices that pass down the pipeline
- The CTM is defined in the user program and loaded into a transformation unit



CTM operations

- The CTM can be altered either by loading a new CTM or by postmutiplication
  - Load an identity matrix: $\mathbf{C} \leftarrow \mathbf{I}$
  - Load an arbitrary matrix: $\mathbf{C} \leftarrow \mathbf{M}$

  - Load a translation matrix: $\mathbf{C} \leftarrow \mathbf{T}$
  - Load a rotation matrix: $\mathbf{C} \leftarrow \mathbf{R}$
  - Load a scaling matrix: $\mathbf{C} \leftarrow \mathbf{S}$

  - Postmultiply by an arbitrary matrix: $\mathbf{C} \leftarrow \mathbf{CM}$
  - Postmultiply by a translation matrix: $\mathbf{C} \leftarrow \mathbf{CT}$
  - Postmultiply by a rotation matrix: $\mathbf{C} \leftarrow \mathbf{C\,R}$
  - Postmultiply by a scaling matrix: $\mathbf{C} \leftarrow \mathbf{C\,S}$

Rotation about a Fixed Point

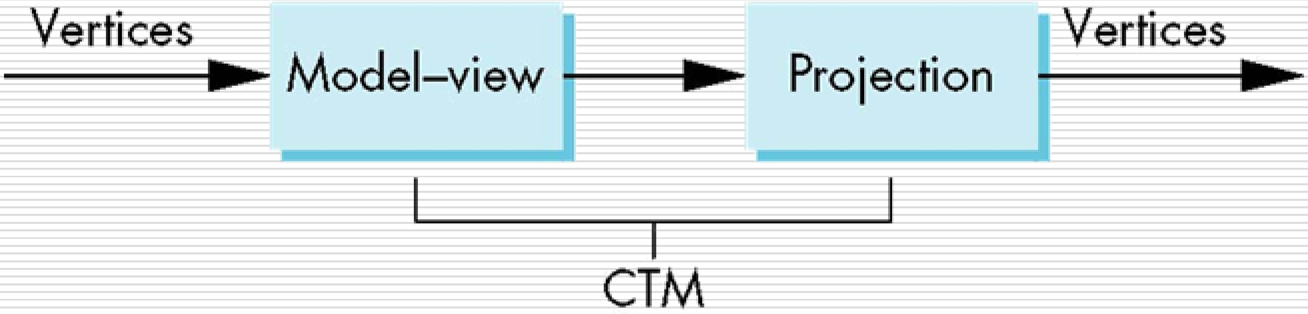
- Start with identity matrix: $\mathbf{C} \leftarrow \mathbf{I}$
- Move fixed point to origin: $\mathbf{C} \leftarrow \mathbf{CT}^{-1}$
- Rotate: $\mathbf{C} \leftarrow \mathbf{CR}$
- Move fixed point back: $\mathbf{C} \leftarrow \mathbf{CT}$

- Result: $\mathbf{C} = \mathbf{T}^{-1}\mathbf{RT}$

- Each operation corresponds to one function call in the program.
- Note that the last operation specified is the first executed in the program.

CTM in OpenGL

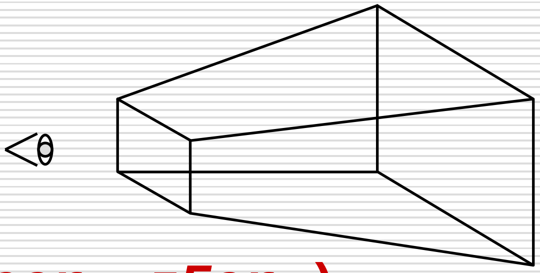
- OpenGL has a model-view and a projection matrix in the pipeline which are concatenated together to form the CTM
- Can manipulate each by first setting the matrix mode



Matrix Operations

- Specify Current Matrix Stack
  glMatrixMode(*GL_MODELVIEW* or *GL_PROJECTION*)
- Other Matrix or Stack Operations
  glLoadIdentity()
  glPushMatrix()
  glPopMatrix()
- Viewport
  - usually same as window size
  - viewport aspect ratio should be same as projection transformation or resulting image may be distorted
    glViewport(*x, y, width, height*)

Projection Transformation



- Shape of viewing frustum
- Perspective projection

```
gluPerspective( fovy, aspect, zNear, zFar )
glFrustum( left, right, bottom, top, zNear, zFar )
```

- Orthographic parallel projection

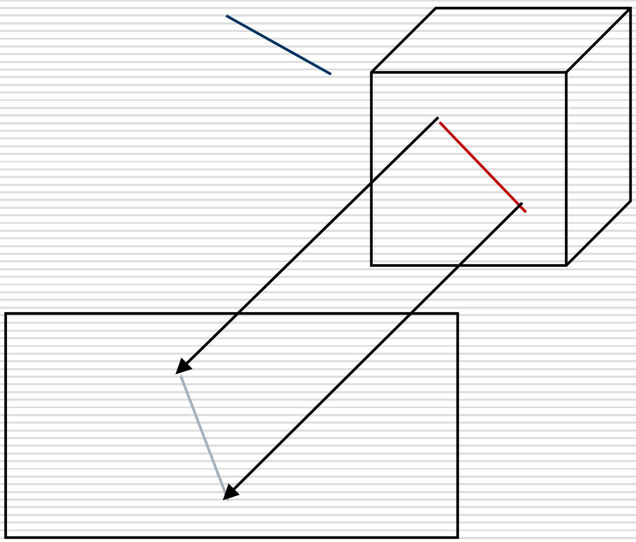
```
glOrtho( left, right, bottom, top, zNear, zFar )
gluOrtho2D( left, right, bottom, top )
```

  - calls glOrtho with z values near zero

Applying Projection Transformations

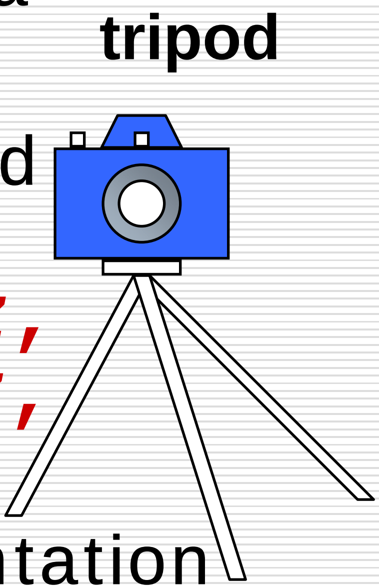
- Typical use (orthographic projection)

```
glMatrixMode( GL_PROJECTION );
glLoadIdentity();
glOrtho( left, right, bottom, top, zNear, zFar );
```

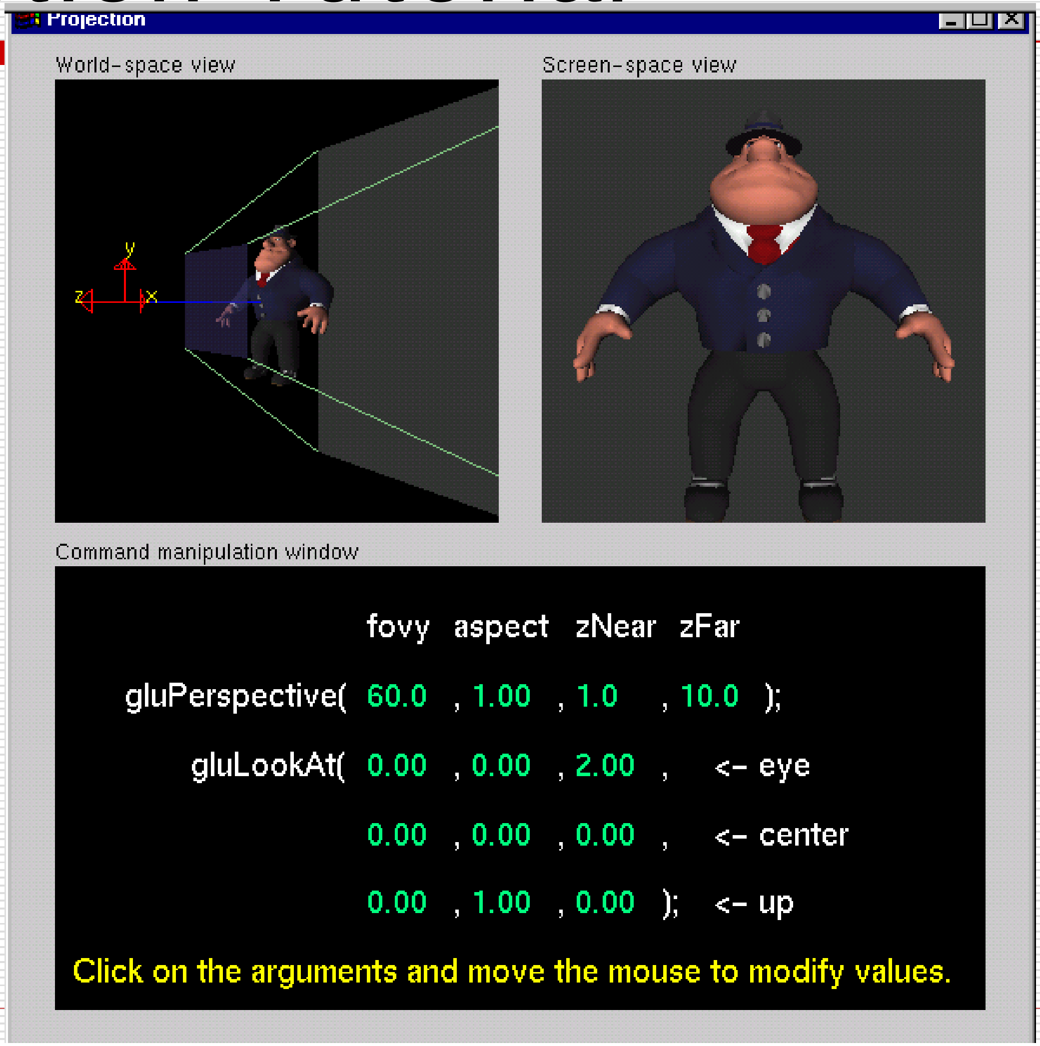


Viewing Transformations

- Position the camera/eye in the scene
  - place the tripod down; aim camera
- To "fly through" a scene
  - change viewing transformation and redraw scene
- **gluLookAt(*eyex, eyey, eyez, aimx, aimy, aimz, upx, upy, upz*)**
  - up vector determines unique orientation
  - careful of degenerate positions



Projection Tutorial



Modeling Transformations

- Move object

  `glTranslate{fd}(` ***`x, y, z`*** **`)`**

- Rotate object around arbitrary axis $\begin{pmatrix} x & y & z \end{pmatrix}$

  `glRotate{fd}(` ***`angle, x, y, z`*** **`)`**

  - angle is in degrees

- Dilate (stretch or shrink) or mirror object

  `glScale{fd}(` ***`x, y, z`*** **`)`**

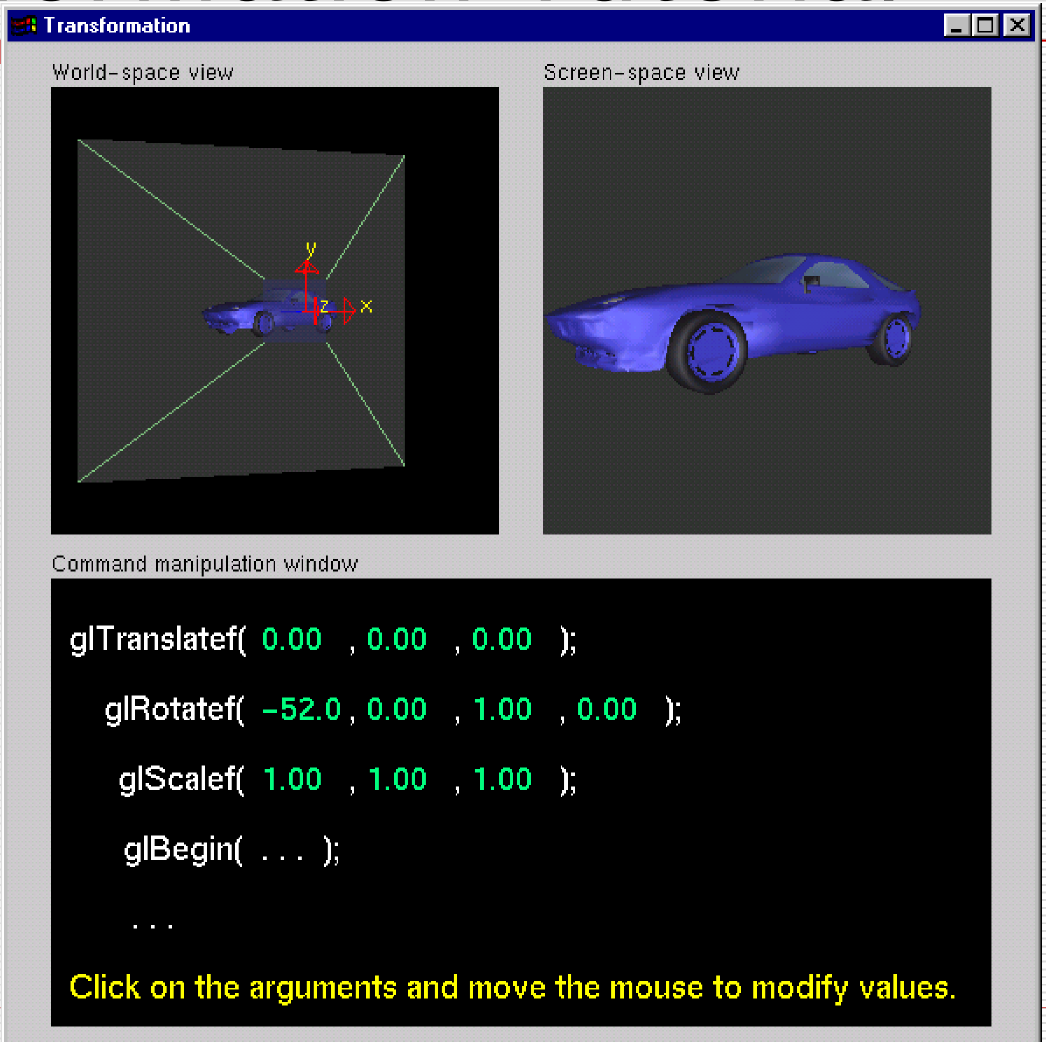
Example

- Rotation about z axis by 30 degrees with a fixed point of (1.0, 2.0, 3.0)

```
glMatrixMode(GL_MODELVIEW);
glLoadIdentity();
glTranslatef(1.0, 2.0, 3.0);
glRotatef(30.0, 0.0, 0.0, .10);
glTranslatef(-1.0, -2.0, -3.0);
```

- Remember that last matrix specified in the program is the first applied

Transformation Tutorial



Arbitrary Matrices

- Can load and multiply by matrices defined in the application program
  - **glLoadMatrixf(m)**
  - **glMultMatrixf(m)**
- The matrix **`m`** is a one dimension array of 16 elements which are the components of the desired 4 x 4 matrix stored by columns
- In **`glMultMatrixf`**, **`m`** multiplies the existing matrix on the right

Matrix Stacks

- In many situations we want to save transformation matrices for use later
  - Traversing hierarchical data structures
  - Avoiding state changes when executing display lists
- OpenGL maintains stacks for each type of matrix
  - Access present type (as set by **`glMatrixMode)`** by
    - **glPushMatrix()**
    - **glPopMatrix()**

Reading Back Matrices

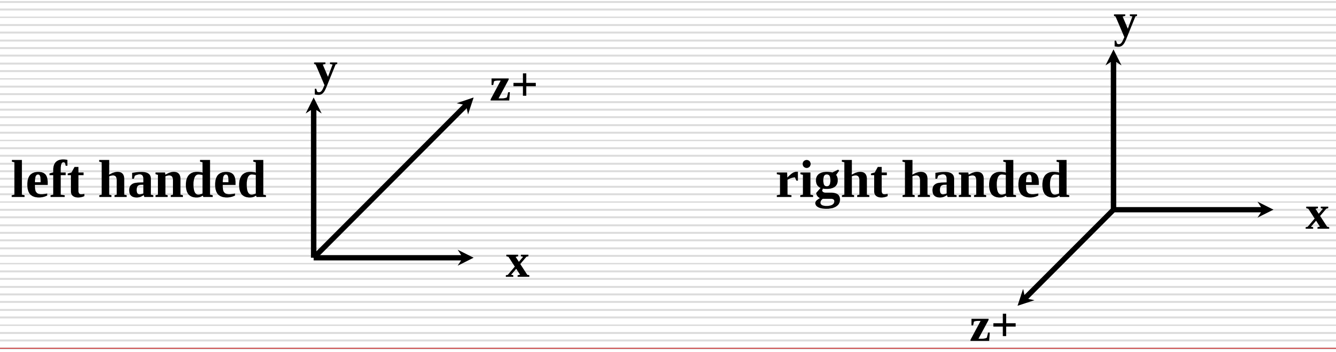
- Can also access matrices (and other parts of the state) by *enquiry (query)* functions
  - **glGetIntegerv**
  - **glGetFloatv**
  - **glGetBooleanv**
  - **glGetDoublev**
  - **glIsEnabled**
- For matrices, we use as
  - **double m[16];**
  - **glGetFloatv(GL_MODELVIEW, m);**

Connection: Viewing and Modeling

- Moving camera is equivalent to moving every object in the world towards a stationary camera
- Viewing transformations are equivalent to several modeling transformations
  - **`gluLookAt()`** has its own command
  - can make your own
    polar view or *pilot view*

Projection is left handed

- Projection transformations (`gluPerspective`, `glOrtho`) are left handed
  - think of ***zNear*** and ***zFar*** as distance from view point
- Everything else is right handed, including the vertexes to be rendered



Common Transformation Usage

- 3 examples of **`resize()`** routine
  - restate projection & viewing transformations
- Usually called when window resized
- Registered as callback for **`glutReshapeFunc()`**

resize():
Perspective & LookAt

```
void resize( int w, int h )
{
   glViewport( 0, 0, (GLsizei) w, (GLsizei) h );
   glMatrixMode( GL_PROJECTION );
   glLoadIdentity();
   gluPerspective( 65.0, (GLdouble) w / h,
                   1.0, 100.0 )
   glMatrixMode( GL_MODELVIEW );
   glLoadIdentity();
   gluLookAt( 0.0, 0.0, 5.0,
              0.0, 0.0, 0.0,
              0.0, 1.0, 0.0 );
}
```

resize():
Perspective & Translate

Same effect as previous LookAt

```
void resize( int w, int h )
{
   glViewport( 0, 0, (GLsizei) w, (GLsizei) h );
   glMatrixMode( GL_PROJECTION );
   glLoadIdentity();
   gluPerspective( 65.0, (GLdouble) w/h,
                   1.0, 100.0 );
   glMatrixMode( GL_MODELVIEW );
   glLoadIdentity();
   glTranslatef( 0.0, 0.0, -5.0 );
}
```

resize():
Ortho (part 1)

```
void resize( int width, int height )
{
   GLdouble aspect = (GLdouble) width / height;
   GLdouble left = -2.5, right = 2.5;
   GLdouble bottom = -2.5, top = 2.5;
   glViewport( 0, 0, (GLsizei) w, (GLsizei) h );
   glMatrixMode( GL_PROJECTION );
   glLoadIdentity();
```

… continued …

resize():
Ortho (part 2)

```
    if ( aspect < 1.0 ) {
        left /= aspect;
        right /= aspect;
    } else {
        bottom *= aspect;
        top *= aspect;
    }
    glOrtho( left, right, bottom, top, near,
    far );
    glMatrixMode( GL_MODELVIEW );
    glLoadIdentity();
}
```

Compositing Modeling Transformations

- Problem 1: hierarchical objects
  - one position depends upon a previous position
  - robot arm or hand; sub-assemblies
- Solution 1: moving local coordinate system
  - modeling transformations move coordinate system
  - post-multiply column-major matrices
  - OpenGL post-multiplies matrices

Compositing Modeling Transformations

- Problem 2: objects move relative to absolute world origin
  - my object rotates around the wrong origin
    - make it spin around its center or something else
- Solution 2: fixed coordinate system
  - modeling transformations move objects around fixed coordinate system
  - pre-multiply column-major matrices
  - OpenGL post-multiplies matrices
  - must <u>reverse order of operations</u> to achieve desired effect

Additional Clipping Planes

- At least 6 more clipping planes available
- Good for cross-sections
- Modelview matrix moves clipping plane $Ax + By + Cz + D < 0$ clipped

```
glEnable( GL_CLIP_PLANEi )
glClipPlane( GL_CLIP_PLANEi, GLdouble* coeff )
```

Reversing Coordinate Projection

- Screen space back to world space

```
glGetIntegerv( GL_VIEWPORT, GLint viewport[4] )
glGetDoublev( GL_MODELVIEW_MATRIX,
              GLdouble mvmatrix[16] )
glGetDoublev( GL_PROJECTION_MATRIX,
              GLdouble projmatrix[16] )
gluUnProject( GLdouble winx, winy, winz,
              mvmatrix[16], projmatrix[16],
              GLint viewport[4],
              GLdouble *objx, *objy, *objz )
```

- **`gluProject`** goes from world to screen space

Smooth Rotation

- From a practical standpoint, we are often want to use transformations to move and reorient an object smoothly
  - Problem: find a sequence of model-view matrices $\mathbf{M}_0, \mathbf{M}_1, \ldots, \mathbf{M}_n$ so that when they are applied successively to one or more objects we see a smooth transition
- For orientating an object, we can use the fact that every rotation corresponds to part of a great circle on a sphere
  - Find the axis of rotation and angle
  - Virtual trackball

Incremental Rotation

- Consider the two approaches
  - For a sequence of rotation matrices $\mathbf{R}_0, \mathbf{R}_1, \ldots\ldots, \mathbf{R}_n$, find the Euler angles for each and use $\mathbf{R}_i = \mathbf{R}_{iz} \mathbf{R}_{iy} \mathbf{R}_{ix}$
    - Not very efficient
  - Use the final positions to determine the axis and angle of rotation, then increment only the angle
- Quaternions can be more efficient than either

Quaternions

- Extension of imaginary numbers from 2 to 3 dimensions
- Requires one real and three imaginary components **i**, **j**, **k**

  - $q = q0 + q1\mathbf{i} + q2\mathbf{j} + q3\mathbf{k} = [\mathbf{w}, \mathbf{v}]$; $\mathbf{w} = q0$, $\mathbf{v} = (q1, q2, q3)$
  - where $\mathbf{i}^2 = \mathbf{j}^2 = \mathbf{k}^2 = \mathbf{ijk} = -1$
  - **w** is called **scalar** and **v** is called **vector**

- Quaternions can express rotations on sphere smoothly and efficiently. Process:
  - Model-view matrix $\rightarrow$ Quaternion
  - Carry out operations with Quaternions
  - Quaternion $\rightarrow$ Model-view matrix

Basic Operations Using Quaternions

- Addition
  - $q + q' = [w + w', v + v']$
- Multiplication
  - $q \bullet q' = [w \bullet w' - v \bullet v', v \times v' + w \bullet v' + w' \bullet v]$
- Conjugate
  - $q^* = [w, -v]$
- Length
  - $|q| = (w^2 + |v|^2)^{1/2}$
- Norm
  - $N(q) = |q|^2 = w^2 + |v|^2 = w^2 + x^2 + y^2 + z^2$
- Inverse
  - $q^{-1} = q^* / |q|^2 = q^* / N(q)$
- Unit Quaternion
  - q is a unit quaternion if $|q| = 1$ and then $q^{-1} = q^*$
- Identity
  - [1, (0, 0, 0)] (when involving multiplication)
  - [0, (0, 0, 0)] (when involving addition)

Angle and Axis & Eular Angles

- Angle and Axis
  - $q = [\cos(\theta/2), \sin(\theta/2) \cdot v]$
- Eular Angles
  - $q = q_{yaw} \cdot q_{pitch} \cdot q_{roll}$
    - $q_{roll} = [\cos (y/2), (\sin(y/2), 0, 0)]$
    - $q_{pitch} = [\cos (q/2), (0, \sin(q/2), 0)]$
    - $q_{yaw} = [\cos(f /2), (0, 0, \sin(f /2)]$

Matrix-to-Quaternion Conversion

```
MatToQuat (float m[4][4], QUAT * quat) {
        float tr, s, q[4];
        int i, j, k;
        int nxt[3] = {1, 2, 0};
        tr = m[0][0] + m[1][1] + m[2][2];
        if (tr > 0.0) {
                s = sqrt (tr + 1.0);
                quat->w = s / 2.0;
                s = 0.5 / s;
                quat->x = (m[1][2] - m[2][1]) * s;
                quat->y = (m[2][0] - m[0][2]) * s;
                quat->z = (m[0][1] - m[1][0]) * s;
        } else {
                i = 0;
                if (m[1][1] > m[0][0]) i = 1;
                if (m[2][2] > m[i][i]) i = 2;
                j = nxt[i];
                k = nxt[j];
                s = sqrt ((m[i][i] - (m[j][j] + m[k][k])) + 1.0);
                q[i] = s * 0.5;
                if (s != 0.0) s = 0.5 / s;
                q[3] = (m[j][k] - m[k][j]) * s;
                q[j] = (m[i][j] + m[j][i]) * s;
                q[k] = (m[i][k] + m[k][i]) * s;
                quat->x = q[0];
                quat->y = q[1];
                quat->z = q[2];
                quat->w = q[3];
        }
}
```

Quaternion-to-Matrix Conversion

```
QuatToMatrix (QUAT * quat, float m[4][4]) {
    float wx, wy, wz, xx, yy, yz, xy, xz, zz, x2, y2, z2;
    x2 = quat->x + quat->x; y2 = quat->y + quat->y;
    z2 = quat->z + quat->z;
    xx = quat->x * x2; xy = quat->x * y2; xz = quat->x * z2;
    yy = quat->y * y2; yz = quat->y * z2; zz = quat->z * z2;
    wx = quat->w * x2; wy = quat->w * y2; wz = quat->w * z2;
    m[0][0] = 1.0 - (yy + zz); m[1][0] = xy - wz;
    m[2][0] = xz + wy; m[3][0] = 0.0;
    m[0][1] = xy + wz; m[1][1] = 1.0 - (xx + zz);
    m[2][1] = yz - wx; m[3][1] = 0.0;
    m[0][2] = xz - wy; m[1][2] = yz + wx;
    m[2][2] = 1.0 - (xx + yy); m[3][2] = 0.0;
    m[0][3] = 0; m[1][3] = 0;
    m[2][3] = 0; m[3][3] = 1;
}
```

SLERP-Spherical Linear intERPolation

- Interpolate between two quaternion rotations along the shortest arc.

- $$\mathrm{SLERP}(p,q,t) = \frac{p \bullet \sin((1-t)\bullet\theta) + q \bullet \sin(t \bullet \theta)}{\sin(\theta)}$$
  - where $\cos(\theta) = w_p \bullet w_q + v_p \bullet v_q$
    $= w_p \bullet w_q + x_p \bullet x_q + y_p \bullet y_q + z_p \bullet z_q$

- If two orientations are too close, use linear interpolation to avoid any divisions by zero.